



ANANDA COLLEGE COLOMBO 10

10 E I

Final Term Test - 2022 January
G.C.E. (A/L) Examination - 2022 February

Grade 13

Combined Mathematics - I

Time : 3 hours

Name :

Grade :

For candidate's use only:

- * Answer **all** questions in **Part A** and only five questions in **Part B**
- * Answer all questions in **Part A** with the space provided

For marking examiner's use only.

Part A

Questions Numbers	Marks
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
Total	

Part B

Question Numbers	Marks
1	
2	
3	
4	
5	
6	
7	
Total	

	Marks
Part A	
Part B	
Total	

- (1) Using the principle of mathematical induction prove that $\frac{n^3}{3} + \frac{2n}{3}$ is an integer for $n \in \mathbb{Z}^+$

- (2) Sketch the graphs of $y = |2x - 5|$ and $y = |6x + 1| - 3$ in the same diagram. Hence find all the real values of x satisfying the inequality $|3x + 1| - |x - 5| < 3$.

(5) Show that $\lim_{x \rightarrow 0} \frac{x^4 \tan 2x}{[\sqrt{x+5}-\sqrt{5}] [1-\cos 2x]^2} = \sqrt{5}$

- (6) Sketch the graphs of $y = x^2 - 4x + 7$ and $y = -x^2 + 4x - 7$ on the same diagram. Hence find the volume of the solid enclosed by the curves $y = x^2 - 4x + 7$, $y = -x^2 + 4x - 7$ and the two lines $x = 0$ and $x = 4$ rotated about the x-axis through π radians.

- (7) The curve C is given by the parametric equation, $x = 5 \cos \theta$ and $y = 4 \sin \theta$. The normal drawn to the curve at the point $\theta = \frac{\pi}{4}$ meets the curve again at the point $\theta = \alpha$. Show that $16\sqrt{2} \sin \alpha - 25\sqrt{2} \cos \alpha + 9 = 0$.

- (8) Write down the equation of the circle $S_1 = 0$ where its centre is the origin and the radius is 5 units. Write down the equation of the circle $S_2 = 0$ which contact both x-axis and y-axis and lies in the first quadrant. Find the equation of the circle which passes through the intersection points of $S_1 = 0$, $S_2 = 0$ and also its centre lies on the common chord of the above two circles.

- (10) Solve $\tan^{-1}\left(\frac{1}{2x+1}\right) + \tan^{-1}\left(\frac{1}{4x+1}\right) = \tan^{-1}\left(\frac{2}{x^2}\right)$

Part B

Answer five questions only.

11. (a) Let α and β are the roots of the quadratic equation $ax^2 + 2bx + c = 0$ and $\alpha + h$, $\beta + h$ are the roots the quadratic equation $px^2 + 2qx + r = 0$. Show that $\frac{b^2 - ac}{q^2 - pr} = \frac{a^2}{p^2}$.
- $f(x) = x^2 + x + A$ and $g(x) = x^2 - 5x + B$ are two quadratic functions. Roots of the equation $g(x) = 0$ are greater by constant K each than the roots of the equation $f(x) = 0$. Furthermore if one root of the equation $g(x) = 0$ is four times than the other root then find the roots of $f(x)$ and $g(x)$. Find the value of the constants A and B . State the two functions $f(x)$ and $g(x)$.
- (b) Let $f(x) = x^3 + ax^2 + (5 - k^2)x + b + 2k^2 = 0$, here $a, b \in \mathbb{R}$ and K is a constant. $(x-2)$ is a factor of $f(x)$. When $f(x)$ is divided by $(x-1)$ the remainder is $k^2 - 36$. Find the values of a and b . Express $f(x)$ as a product of linear factors. For every real values of $x > 2$ find the range of values of k such that $f(x) > 0$.
12. (a) A society has to be formed among the employers of a company including the staff of it's head office and other 6 branches. In order to select the board of officials for the committee each of the seven branches including the head office propose 3 employers from each of the five divisions A, B, C, D, E. Among them 5 individuals has to be selected as the board of officials. Calculate the number of different committees that can be formed in each of the following instances.
- Officials representing one from each division.
 - Officials that does not includes more than one from each branch including the head office.
 - Compulsory to include two employers from the head office for the board of committee.
 - Selecting officials for the committee without two specific branches and including only from two divisions out of the five divisions.
- (b) 7, 19, 37, lies on a sequence where the r^{th} term is given by a quadratic expression in r . Find the r^{th} term of the sequence.
- Write down the r^{th} term u_r of the series.
- $$\frac{7}{(1^3 + 1)(2^3 + 1)} + \frac{19}{(2^3 + 1)(3^3 + 1)} + \frac{37}{(3^3 + 1)(4^3 + 1)} + \dots$$
- Find the function $f(r)$ such that $U_r = f(r) - f(r+1)$.
- Find $\sum_{r=1}^n u_r$.
- Give reasons whether the infinite series $\sum_{r=1}^n U_r$ is convergent when $n \rightarrow \infty$
- Find the summation of $\sum_{r=1}^n U_r$ when $n \rightarrow \infty$
- Furthermore show that $\sum_{r=1}^{2n} U_r - \sum_{r=1}^n U_r = \left[\frac{1}{(n+1)^3 + 1} \right] - \left[\frac{1}{(2n+1)^3 + 1} \right]$.

13. (a) Let $A = \begin{pmatrix} 1 & 2 & -3 \\ 4 & 0 & 5 \end{pmatrix}$, $B = \begin{pmatrix} 2 & -1 & 1 \\ 1 & 2 & 0 \end{pmatrix}$, $C = \begin{pmatrix} -5 & 10 \\ 1 & 3 \end{pmatrix}$ are three matrices.

Find the matrix D such that $D = B A^T - C$. If $f(x) = x^2 - 4x + 3$ find $f(D)$

Take $f(D) = E$,

- (i) Write down the matrix E as a sum of the symmetric and skew symmetric matrices.
 (ii) Find E^{-1} .

- (b) Z, Z_1 and Z_2 are three complex numbers. Prove that,

(i) $|Z|^2 = Z \bar{Z}$

(ii) $|Z_1 \cdot Z_2| = |Z_1| |Z_2|$

- (iii) Z_1, Z_2, Z_3, Z_4 are the vertices of a rhombus $ABCD$ taken in the clockwise direction and $\angle CBA = \frac{2\pi}{3}$.

Find the value of $\frac{Z_3 - Z_2}{Z_1 - Z_2}$.

Show that $2\sqrt{3} Z_2 = (\sqrt{3} - i) Z_1 + (\sqrt{3} + i) Z_3$.

- (c) Express $Z = \frac{1+7i}{(2-i)^2}$ in the form of $r(\cos\theta + i\sin\theta)$ where $r > 0$, $\frac{\pi}{2} < \theta < \pi$ and determine the values of r and θ . By using De Moivre's theorem find the value of $\frac{17}{Z^3}$. Deduce the

value of $\left(Z^{\frac{17}{3}} - \frac{1}{Z^{\frac{17}{3}}} \right)^2$.

14. (a) Let $f(x) = \frac{(x-3)(x+1)}{x(x-2)}$ for $x \neq 0$ and $x \neq 2$.

Determine the values of A and B such that $f(x) = A + \frac{B}{x(x-2)}$. Hence obtain the first derivative

of $f(x)$ as $f'(x)$. Then calculate the coordinates of the turning point and determine its nature.

Express the increasing and decreasing intervals of $f(x)$ separately.

Find the second derivative of $f(x)$ and determine whether there are points of inflection for the function. Hence obtain the concavity of the function in each interval.

Sketch the graph of the function $y = f(x)$ by indicating asymptotes, turning points and the points of inflection if they exist.

- (b) An equilateral triangle and a rectangle should be made by using a 30 meter long wire such that the length of the rectangle is three times that of the breadth. Find the length of a side of a rectangle such that the area of the composite rectangle and the triangle is minimum.

15. (a) Using the substitution $x = \frac{1}{2}(1 + \sin \theta)$ show that,

$$\int_{1/4}^{3/4} \frac{x}{\sqrt{x-x^2}} dx = \frac{1}{2} \int_{-\pi/6}^{\pi/6} (1 + \sin \theta) d\theta$$

Hence evaluate $\int_{1/4}^{3/4} \frac{x}{\sqrt{x-x^2}} dx$.

- (b) Using the integration by parts show that,

$$\int_0^{2\pi} e^x \cos\left(\frac{x}{2} + \frac{\pi}{4}\right) dx = \frac{-3\sqrt{2}}{2} (e^{2\pi} + 1)$$

- (c) Let $I = \int_0^a \frac{f(x)}{f(x) + f(a-x)} dx$. By using suitable substitution show that $I = \frac{a}{2}$.

Hence find the value of followings.

(i) $\int_0^1 \frac{\ln(x+1)}{\ln(2+x-x^2)} dx$

(ii) $\int_0^{\pi/2} \frac{\sin x}{\sin(x + \frac{\pi}{4})} dx$

16. (a) Show that the perpendicular distance from the point (x_1, y_1) to the straight line $ax+by+c=0$ is,

$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

- (b) State the sufficient requirement for the two circles $S_1: x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$ and $S_2: x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$ to be intersect each other orthogonally.

- (c) Find the relative position of the two circles $S_1 \equiv x^2 + y^2 - 8x - 2y + 16 = 0$ and $S_2 \equiv x^2 + y^2 - 4x - 4y - 1 = 0$. Find the equations of the common tangents that can be drawn to the above two circles. Another circle S which passes through the origin intersect above two circles $S_1 = 0$ and $S_2 = 0$ orthogonally. Find the equation of S .
Find the equations of the common chords of above each pair of circles S_1 and S_2 , S and S_1 , S and S_2 .
Show that all the three common chords are concurrent.

17. (a) If $\tan x + \tan(x + \frac{\pi}{3}) + \tan(x + \frac{2\pi}{3}) = 3$ then show that $\tan 3x = 1$.

- (b) State the sine rule and cosine rule for any triangle ABC in standard notation.

Let CD be the median line of the triangle ABC. If $\hat{ADC} = \alpha$ and the area of the triangle ABC is Δ , then show that $b \sin A = a \sin B$ and

$$\frac{2c \cos \alpha}{(a^2 - b^2)} = \frac{c \sin \alpha}{2\Delta} = \frac{1}{CD}$$

- (c) Let $f(x) = \sin^4 x + \cos^2 x$; $x \in \mathbb{R}$. State $f(x)$ in the form of $A + B \cos kx$. Here A, B and k are constants to be determined.

Furthermore deduce that $\frac{3}{4} \leq f(x) \leq 1$.

Sketch the graph of $y = f(x)$ in the range $-\frac{\pi}{2} \leq x \leq \frac{\pi}{3}$.



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Grade 13

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Part A

Questions Numbers	Marks
1	
2	
3	
4	
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6	
7	
8	
9	
10	
Total	

Part B

Question Numbers	Marks
1	
2	
3	
4	
5	
6	
7	
Total	

	Marks
Part A	
Part B	
Total	

Part A

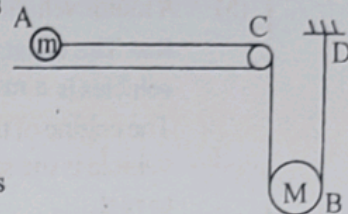
Answer all questions.

- (1) Two particles A and B of masses $2m$ and $3m$ collided directly in the opposite directions in the same straight line with velocities $2u$ and u respectively. Taking e as the coefficient of restitution between the particles, Find the velocity of the particle B and show that it rebounds. Show also that when $e = \frac{1}{9}$ the particle A comes to rest.

- (2) The position vector of a particle moving on a plane at time ' t ' is given by $\underline{r} = 4t \underline{i} + (t^4 - 32t) \underline{j}$. where $t > 0$. Time is measured in seconds and displacement is in meters.

- (i) Find the velocity and the acceleration of the particle in vector form.
(ii) After what time at which the velocity becomes perpendicular to the acceleration? At this moment, find the displacement of the particle from the origin in vector form.

- (3) One end of a light inextensible string is attached to the particle A of mass m , placed on a rough horizontal plane with the coefficient of friction μ and the string passes over a fixed smooth pulley and the remaining part of the string passes underneath of a moving pulley of mass M . The system releases gently from rest. Find the acceleration of the particle A. Show that the requirement the system to be moved is $\frac{M}{m} > 2\mu$



- (4) A particle P of mass m is attached to a point of a light inextensible string. One end of it is attached to a fixed point A on a vertical smooth wire and other the end connected to a small ring of mass $2m$, below the point A, passing through the vertical wire. Consider that the string can easily be rotated at the point A. When the particle P rotates about the wire with angular velocity ω , $AQ = \ell$, $\hat{APQ} = 90^\circ$, $\hat{QAP} = 30^\circ$. Draw a force diagram indicating all the forces on P and Q. Write sufficient equations to find ω and the tensions of the strings and find the tensions,

- (5) A motor vehicle of mass M kg moves up along an inclined road, angle θ to the horizontal with power P Kw. The constant resistant for the motion is R . At a particular movement the acceleration of the vehicles is $a \text{ ms}^{-2}$, find an expression for the velocity at this moment in terms of P , R , a , M and θ . The engine of the vehicle is switched off at the above movement and considering the resistant to the vehicle is the same above find the deceleration of the vehicle and the time at which the vehicle comes to rest.

- (6) An uniform rod AB of weight W and length $2a$ placed by one end A kept on a smooth horizontal plane and supported by a rough peg C with coefficient of friction $\frac{1}{2}$ where $BC = \frac{a}{2}$. The rod is kept in limiting equilibrium to move upward by horizontal force P on the same vertical plane at the point A. Show that P is given by $\frac{W}{6}(2\sin 2\theta + \cos \theta + 1)$. Where θ is the angle inclined by the rod to the horizontal.

- (9) The probabilities of solving a problem by three students A, B and C separately are $\frac{1}{5}$, $\frac{4}{7}$ and $\frac{3}{8}$ respectively. If the problem is attempted by all students to solve simultaneously, find the probabilities of exactly one of them solving it.

- (10) The mean and standard deviation of a set of 10 numbers are 8 and 6 respectively. Later it was found that the number 9 was misread as 5. Find the correct mean and variance of the set of numbers.



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Part B

11. (a) Two trains X and Y travelling in the same direction on two parallel railway tracks passes through the station A with velocities $2u$ and $3u$ at the same instance. Accelerations of these two are $3f$ and $2f$ respectively. If after a time " t_1 ", train X overtakes train Y at the point B, show

$$\text{that, } t_1 = \frac{2u}{f}$$

Find the velocities of these two trains at this moment. After that train Y travels with obtained constant velocity and train X with a retardation f . If the train Y overtakes train X at the point C after period of time t_2 , find the velocity of train X at this moment.

Show that the distance between A and C is $\frac{24u^2}{f}$.

- (b) A ship is sailing at a constant speed of $u \text{ kmh}^{-1}$ along a straight course. The closest distance from the point A of the port and the path of the ship is ' a ' km. When the ship is at a distant $b (> a)$ km from the port before it reached to point A a boat starts to sail from the port to catch the ship. Prove that the minimum constant velocity that the boat should have to reach the ship is $\frac{au}{b} \text{ kmh}^{-1}$. Sketch the path of the boat.

If the boat can travel with a speed $v \text{ kmh}^{-1}$ $\left(u > v > \frac{au}{b}\right)$ draw the velocity triangles in the

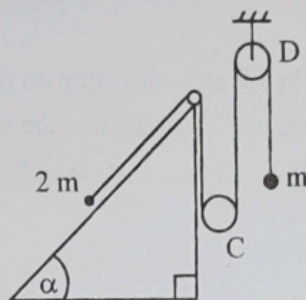
same diagram and hence show that the time differ by $\frac{2\sqrt{b^2 v^2 - a^2 u^2}}{u^2 - v^2}$ hours to come to these

two places.

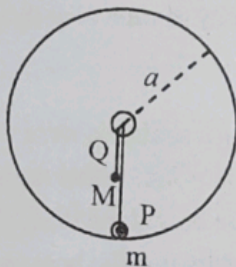
12. (a) A light inextensible string passes over a smooth fixed pulley, connected to an fixed inclined plane α to the horizontal and passes underneath of a moving pulley C and the remaining portion of the string passes over a fixed pulley D. One end of the string connected to mass $2m$, lying on the inclined plane and the other end is connected to a mass m as shown in the diagram. Write sufficient equations in order to find the accelerations of the masses $2m$, m and the moving pulley and the tension of the string.

Hence show that the tension of the string is $\frac{10m}{23}(3 + \sin \alpha)g$.

Further more show that in this motion the pulley C does not comes to rest at anytime. But show that the mass $2m$ comes to rest for several values of α and determine those values of α .



(b)



A smooth bead P of mass m passes through a vertically fixed smooth circular wire. A light inextensible string passes through a smooth ring fixed at the centre of the wire. One end is connected to the bead and the other end tied to a particle Q of mass M . At the start, the bead places at the bottom of the wire and is projected horizontally with velocity $\sqrt{kg a}$ ($k > 1$) in order to move a circular motion

along the wire. When the string connected to the bead makes an acute angle θ , downward vertical, show that the velocity of the bead V is given by $V^2 = kga - 2ga + 2ga \cos \theta$. Show that the reaction R on the bead by the wire is given by $mg(k - 2 + 3 \cos \theta - \frac{M}{m})$. By taking $k = 6$, if $m < M < 7m$, show that at a particular moment this reaction vanishes.



13. One end of a light inextensible string is attached to a particle A, placed at O, on a horizontal smooth table and the string passes over a smooth pulley which is fixed at the edge of the table and the other end is attached to a particle B which hangs touching the pulley as shown in the diagram. Another light extensible string of natural length ℓ and coefficient of elasticity mg connected with one end to O and the other end to the particle A. The system releases from rest with the inextensible string taut. Find the velocity of A just before the extensible string is taut. When the length of the elastic string is $x(>\ell)$ from O, show that the motion of A is satisfied the equation $\ddot{x} = \frac{g}{2\ell}(x - 2\ell)$.

Taking $X = x - 2\ell$ show that the above equation can be written $\ddot{X} + \omega^2 X = 0$. By assuming $\dot{X}^2 = \omega^2(a^2 - X^2)$. Find the amplitude 'a' of the motion.

If the particle A slightly reaches the pulley and B reaches its deepest point C, find the verticle height of B from the pulley. Find the time elapses the particle B reaches to C. Now when B reaches the point C another particle of mass m is coleases with B gently. Thereafter assuming that the motion

takes place upwords, show that its motion is satisfied by the equation, $\ddot{y} + \frac{g}{3\ell}(y - 3\ell) = 0$, where y is OA.

Futhermore show that this motion is entirely a simple harmonic motion. Show that the total time elapsed for the particle A until it comes to an instantaneous rest at the second time is

$$\sqrt{\frac{\ell}{g}} \left(2 + (\sqrt{3} + \sqrt{2})\pi - \sqrt{2} \tan^{-1} \sqrt{2} \right).$$

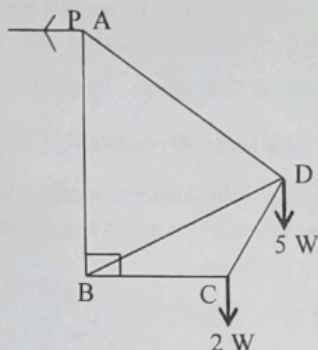
14. (a) (i) Three position vectors $\underline{a}$, $\underline{b}$ and $\underline{c}$ are given by the three sides OA, OB and OC of the rhombus OACB respectively. If the length of a side of the rhombus is ℓ , then show that, $\underline{a} \cdot \underline{c} - \underline{a} \cdot \underline{b} = \ell^2$. If the angle $\hat{A}OB$ of the rhombus is 60° , show that $3\underline{a} \cdot \underline{b} = \underline{a} \cdot \underline{c}$. Show the diagonals of the rhombus are perpendicular to each other.
- (ii) D and E are the mid points of the sides AC and AB of the triangle ABC. If CE and BD intersect at X, then find $\overline{EB}$ and $\overline{BD}$ in terms of $\overline{AB}$ and $\overline{AD}$ and Find the ratio CX : XE.
- (b) The co-ordinates of the points O, A, B and C are given by (0,0), (2a, 0), (2a, a) and (0,a) respectively. Forces of magnitudes F_1 , F_2 , F_3 and F_4 act along the sides OA, AB, BC and CO respectively in the directions indicated by the order of the letters. If the anticlock wise moments around the points O, A, B are G, 2G and 3G respectively and if the magnitude of the resultant is P, then show that the length of AC is $\frac{5G}{2P}$.
- (c) In the rectangle ABCD, the length of sides AB and BC are 4a and 3a respectively. Forces of magnitudes 8p, 7p, 3p, 2p, 10p and 15p act along the sides $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DA}$, $\overline{AC}$ and $\overline{DB}$ respectively in the directions indicated by the order of the letters. Futhermore a couple of magnitude 10 pa acts along the direction of DCBA. Show that the system can be reduced to a single resultant force and hence find the magnitude, direction and the point at which the equation of the line of action intersects AB.

15. (a) Three uniform rods AB, BC and CD each of length $2a$ and weights w , $2w$ and w respectively are smoothly joined at B and C. The rod BC is horizontal and the ends A and D are kept on two smooth planes inclined at an angle α to the horizontal. Two light inextensible strings of equal length are connected to the mid points of rods AB and CD and the other ends are connected to the mid point of rod BC. The strings are kept taut and framework ABCD is in equilibrium in a vertical plane such that $\hat{ABC} = \hat{BCD} = 120^\circ$.

Find,

- the reaction of the rod AB at the end A to the plane.
- the tensions of the strings
- the horizontal and vertical components of the reaction at the joint B and the angle makes with the vertical.

(b)

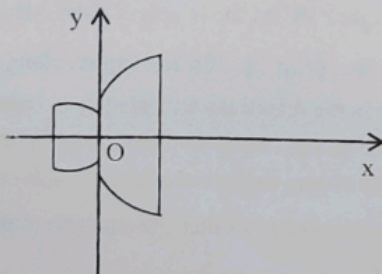


The figure shows a framework consisting of five light rods. ABD is an equilateral triangle.

$\hat{ABC} = 90^\circ$ and $BC = CD$. It is smoothly hinged to a rigid point at B. Weights $2w$ and $5w$ are suspended at C and D respectively and the framework is held in equilibrium with AB vertical by means of a horizontal force p applied at A. Find the value of the force p and draw a stress diagram using Bow's notation. Hence find the stresses in five rods stating whether they are tensions or thrusts.

16. In a hollow sphere of radius r , a ring is separated by cutting through vertical planes, which are distances $r \cos \alpha$ and $r \cos \beta$; $\beta > \alpha$ from the centre and on the same side of the sphere. If the mass of the ring is m , show that it is given by $2\pi r^2 \sigma (\cos \alpha - \cos \beta)$ and show that the centre of mass of ring from the centre of the sphere is $\frac{a}{2}(\cos \alpha + \cos \beta)$. Where σ is the surface density of the material.

Two such rings with $\alpha = \frac{\pi}{4}$ and $\beta = \frac{\pi}{3}$ and radii r and $\frac{r}{2}$ are joined by keeping a thin disc of radius $\frac{r}{\sqrt{2}}$ and mass $\frac{m}{4}$ is between the common plane as shown in the diagram.



The centres of masses of each body and the point O (centre of the disc) lie on the same axis. Show that,

- (i) the mass of the small ring is $\frac{m}{4}$, when $k = \frac{(2 + \sqrt{2})}{4\sqrt{2}}$
- (ii) the centre of mass of the small ring is $\frac{r}{2} \left(\frac{1}{\sqrt{2}} - k \right)$ from o.
- (iii) the centre of mass of the big ring is $r \left(\frac{1}{\sqrt{2}} - k \right)$ from o.

Also find the centre of mass of the composite body from O. Now the composite body is hung from a point at the edge of the thin disc, find the angle inclined the axis with the vertical.

17. (a) (i) Let, $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Find the probability of getting real pairs of solution for 'p' and 'q' in the quadratic equation $x^2 + px + q = 0$ in the set A.
- (ii) Probability of attendance in the school for the three students K, L, M are $\frac{1}{5}, \frac{1}{4}, \frac{1}{3}$ respectively and the attendance are independent of each other. On a certain day if one of them is present in the school out of these three, find the probability that this student will be L.
- (b) A machine produces iron rods of standard length 10 cm. Following table gives the grouped frequency distribution of difference of the length of rods, 'x', 10^{-3} cm compared to the standard length 10 cm of a rod.

Find the mode, median, mean, standard deviation and the co-efficient of skewness of the above distribution.

What is the shape of the distribution.

X (10^{-3} cm)	No : of rods
(-50) - (-31)	12
(-30) - (-11)	18
(-10) - 9	26
10 - 29	18
30 - 49	14
50 - 69	12

