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Provincial Department of Education - NWP

තෙවන වාර පරීක්ෂණය - 12 ශ්‍රේණිය - 2024

Third Term Test - Grade 12 - 2024

විභාග අංකය: ....

Combined Mathematics - I

Time: 03 Hours

පැය තුනයි
மூன்று மணித்தியாலம்
Three hours

අමතර කියවීම් කාලය - මිනිත්තු 10 යි
மேலதிக வாசிப்பு நேரம் - 10 நிமிடங்கள்
Additional Reading Time - 10 minutes

Use additional reading time to go through the question paper, select the questions you will answer decide which of them you will prioritise.

Index number | | | | | | | |

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  Part A (Questions 1 - 10) and **Part B** (Questions 11 - 17)
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For Examiners' Use only

(10) Combined Mathematics I		
Part	Question No.	Marks
A	1	
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	7	
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B	11	
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	16	
	17	

Total

In numbers	
In words	

PART - A

(01) Find all real values of x satisfying the inequality $\dfrac{2 - x}{3 + x} \leqslant 1$.

(02) Find the constants A and B such that $\dfrac{x + 1}{x^2 - x} = \dfrac{A}{x} + \dfrac{B}{x-1}$. ***Hence or otherwise***, find the partial fractions of $\dfrac{x^3 + 1}{x^2 - x}$.

(03) Show that $\lim\limits_{x \to 0} \dfrac{1 - \cos 3x - x\sin 2x}{x \sin 4x} = \dfrac{5}{8}$.

(04) In the triangle OAB, it is given that at the origin O, $A\hat{O}B = 90^0$ and the area of the triangle is 13 square units. If $A \equiv (-\boldsymbol{a},3)$ and $B \equiv (3\boldsymbol{a}, 2\boldsymbol{a})$ for $\boldsymbol{a} \in \mathbb{R}^+$, Find the value of $\boldsymbol{a}$. Find the coordinates of the point which the line AB cut the Y axis.

(05) If $\left(\frac{1}{3}, -\frac{23}{27}\right)$ and $(1, -1)$ are turning points of the function $f(x) = x^3 - ax^2 + bx - c$ find a and b.

(06) Find all real values of $\boldsymbol{a}$ such that the roots of the quadratic equation $\boldsymbol{a}^2 x^2 - (\boldsymbol{a}+1)x + 1 = 0$ are real. If it is given that the roots of the equation are coincident and $\boldsymbol{a} \in \mathbb{Z}^+$. Find the value of $\boldsymbol{a}$ and the roots of the above quadratic equation.

(07) Let $f(x) = \text{Log}_2 (2x - 3)$; $x > 3/2$

$g(x) = 2x^2 - 6x + K$; $K \in IR$

Find the composite function $f \circ g$.

Find the value of k such that $f \circ g = 2 f(x)$.

(08) The remainder when $f(x) = x^3 + 2x^2 + \boldsymbol{a}x + \boldsymbol{b}$ $\boldsymbol{a}, \boldsymbol{b} \in IR$, $\boldsymbol{a} \neq 0$, is divided by $(x - \boldsymbol{a})$ is b. Show that $\boldsymbol{a} = -3$. For that value of $\boldsymbol{a}$, if the remainder when $f(x)$ is divided by $(x-1)$ is $(\boldsymbol{ab} + 1)$, find the value of $\boldsymbol{b}$.

(09) If $\cot^2\theta + \tan^2\theta = \lambda$, where $\lambda \in \mathbb{R}^+$

Show that $\cos 4\theta = \dfrac{\lambda - 6}{2 + \lambda}$. When $\lambda = 2$, find the general solution of θ.

(10) If it is given that $\tan y = \dfrac{1 + \sqrt{2}\sin\frac{\theta}{2}}{1 + \sqrt{2}\cos\frac{\theta}{2}}$, $\cos\frac{\theta}{2} \neq \frac{1}{\sqrt{2}}$. Show that

$\tan y = \tan\left(\frac{\pi}{8} + \frac{\theta}{4}\right)$. Here if y and $\left(\frac{\pi}{8} + \frac{\theta}{4}\right)$ is in the range $\left[0, \frac{\pi}{2}\right)$ deduce that $\sin 4y = \cos\theta$.

(11) (a) If $f(x) = x^2 - 2ax + 1$ for $a > 1$, show that the roots of $f(x) = 0$ are real and distinct. Sketch the rough graph of $y = f(x)$. If $g(x) = x^2 + 2bx - 1$ for $b \in \mathbb{R}^+$ and if the roots of $g(x) = 0$ are α and β, Write down the expressions for $\alpha + \beta$ and $\alpha\beta$. Hence show that α and β take opposite signs.

Sketch a rough graph of $y = g(x)$ for $0 < \alpha < a$ and $\beta < 0$ in the same coordinate plane.

By considering the sign of $f(\alpha) \times f(\beta)$, show that $b^2 - a^2 + 1 < 0$

(b) Show that the remainder when the polynomial $f(x)$ is divided by $(x - k)^2$ is $f'(k)\,x + f(k) - kf'(k)$. Here $f'(x)$ is the first differential coefficient of $f(x)$; $k \in \mathbb{R}$.

When the polynomial $f(x) = x^4 + 2x^3 + ax^2 + bx + 2$ is divided by $(x-2)^2$, if the remainder is $61x - 86$, find a and b.

(12) (a) If $f(x) = |2x - 3| - x$ and $g(x) = |x - 1|$, sketch the graphs of $y = f(x)$ and $y = g(x)$ in the same coordinate plane. ***Hence or otherwise,***

(i) Find the set of values of x, such that $|2x - 3| - x > |x - 1|$

(ii) Find the set of values of λ for the quadratic equation $|2x - 3| - x = |x - \lambda|$ to have two real and distinct roots.

(iii) Using the graph calculate the set of values of α such that $g(\alpha) - f(\alpha) = 2$

(b) (i) If $x + y + z = 0$, show that $x^3 + y^3 + z^3 = 3xyz$ $(x, y, z \in \mathbb{R})$

(ii) Prove that $\text{Log}_x\, y = \dfrac{\ln\, y}{\ln\, x}$.

x, y, and z are three positive real numbers not equal to 1.

(iii) If it is given that $(\log_y x \log_z x - \log_x x) + (\log_x y \log_z y - \log_y y) + (\log_x z \log_y z - \log_z z) = 0$, show that $xyz = 1$.

(13) (a) Using a suitable geometric method, for $x > 0$ prove that $\lim\limits_{x \to 0} \dfrac{\sin x}{x} = 1$.

Find the constants λ and μ such that $\sin^{-1} x - \cos^{-1} x = \sin^{-1}(\lambda x^2 - \mu)$

Hence or otherwise, evaluate $\lim\limits_{x \to \frac{1}{\sqrt{2}}} \dfrac{\sin^{-1} x - \cos^{-1} x}{4x^4 - 1}$

(b) Find the domain of the function $f(x) = \sqrt{1 - \log_2(x^2 - 7x + 8)}$

(c) Let $f(x) = x^2 + 4x + 8$; $f : \mathbb{R}\,[h, \infty) \rightarrow \mathbb{R}\,[k, \infty]$. Find the real constants h and k for $f^{-1}(x)$ to exist. Sketch the rough graph of $y = f^{-1}(x)$.

(14) (a) Using the first principles, find the derivative of $y = x \sin x$.

Using the above result, differentiating appropriate functions show that

(i) $\frac{d^2 y}{dx^2} = 2\cos x - y$

(ii) $x\frac{d^3 y}{dx^3} + x\frac{dy}{dx} + 2y = 0$

(b) Using a suitable substitution or otherwise, if

$y = \sin^{-1}\left(\frac{2x}{1+x^2}\right) + \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, show that

$\frac{dy}{dx} = \frac{4}{1+x^2}$

If it is given that $y = \pi$, deduce that $\frac{dy}{dx} = 2$.

(c) If $y = \theta \ln\theta$ show that $\frac{dy}{d\theta} = 1 + \ln\theta$.

If it is given that x is a differentiable function of θ show that $\frac{dy}{dx} = (1 + \ln\theta)\frac{d\theta}{dx}$

Considering second differential coefficient show that $\theta\frac{d^2y}{dx^2} = (\theta + y)\frac{d^2\theta}{dx^2} \left(\frac{d\theta}{dx}\right)^2$

(15) (a) Let $f(x) = \frac{\boldsymbol{a}x - 1}{(x - \boldsymbol{b})^2}$, $(\boldsymbol{a}, \boldsymbol{b} \in \mathbb{Z})$, $x \neq \boldsymbol{b}$

Show that $f'(x)$, the derivative of $f(x)$, is given by

$f'(x) = \frac{2 - \boldsymbol{ab} - \boldsymbol{a}x}{(x - \boldsymbol{b})^3}$, $x \neq \boldsymbol{b}$

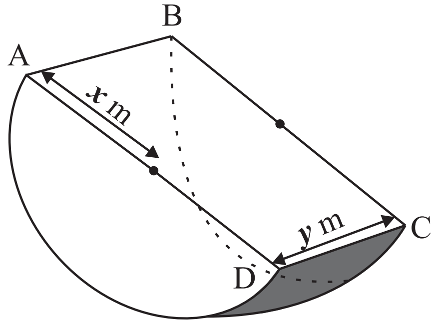
It is given that $f''(x) = \frac{2(\boldsymbol{a}x + 2\boldsymbol{ab} - 3)}{(x - \boldsymbol{b})^4}$; $x \neq \boldsymbol{b}$.

If the curve of $\boldsymbol{y} = f(x)$ has a turning point at $\boldsymbol{x} = -1$ and a point of inflection at $x = -3$, show that $\boldsymbol{a} = 1$ and $\boldsymbol{b} = 3$.

By considering the sign of $f'(x)$, find the intervals on which $f(x)$ is increasing and the intervals on which $f(x)$ is decreasing.

Sketch a rough graph of $y = f(x)$ indicating the turning points, point of inflection and the asymptotes.

(b) Two semicircular plates with each of radius x are kept at a distance y from each other. A rectangular plate is bent to from a curved surface and by fixing them together, a tank with diameter $2x$ and length y is formed as shown in the figure.



It is given that volume of the tank is $8\pi\ m^3$. It the face ABCD is open, show that the total surface area **A** of the tank is $\mathbf{A} = \pi \left[x^2 + \dfrac{16}{x}\right]$.

Find the radius x of the semicircular plate such that the Area **A** of the tank is minimum. Also find the length **y** of the tank.

(16) (a) Using the formula $\sin 3x = 3\sin x - 4\sin^3 x$, deduce that $\cos 3x = 4\cos^3 x - 3\cos x$.

Let $k(x) = \sin^2 x \cos 3x + \cos^2 x \sin 3x$

Using the above results or otherwise, find the constants A, B, C and α such that

$k(x) = \mathrm{A}\cos(x + \alpha)\sin 2x(\mathrm{B}\sin 2x + \mathrm{c})$.

Find the general solution of the equation $k(x) = 0$

(b) In a triangle ABC, the angles A, B, C lie in an arithmetic series such that $\mathrm{A},\ \mathrm{A} + \alpha$, $\mathrm{A} + 2\alpha$. Here $\mathrm{O} < \alpha < \dfrac{\pi}{3}$

Using the sine rule for the triangle ABC,

Show that

(i) $\sin \mathrm{A} = \dfrac{\sqrt{3}\,a}{2b}$

(ii) $\sin\left(\dfrac{\pi}{3} + \alpha\right) = \dfrac{\sqrt{3}\,c}{2b}$.

Using the above results or ***otherwise deduce that*** $\sin\alpha = \dfrac{\sqrt{3}\,(c - a)}{2b}$ and $\cos\alpha = \dfrac{a + c}{2b}$ and show that $a^2 + c^2 - ac = b^2$.

(17) (a) Let $f(x) = \sin^2 x + \sqrt{3}\sin x \cos x + 2\sin x + 3\sqrt{3}\cos x - 3$

Show that $f(x) = (\sin x + 3)\, g(x)$

Here $g(x) = A\sin(x + \beta) - B$. Determine the values of A, B and β and sketch the graph of $y = g(x)$ in the range $[-\pi, \pi]$.

Hence or otherwise, find the value of k or the set of values of k such that the equation

$|g(x)| = k$ has,

(i) one solution.

(ii) two solutions.

(iii) three solutions.

(iv) four solutions, in the range $[-\pi, \pi]$.

(b) Prove that $\tan^{-1}(2x+1) - \tan^{-1}(2x-1) = \tan^{-1}\left(\dfrac{1}{2x^2}\right)$

Hence or otherwise, prove that

$\tan^{-1}\left(\dfrac{1}{2}\right) + \tan^{-1}\left(\dfrac{1}{8}\right) + \tan^{-1}\left(\dfrac{1}{18}\right) + \tan^{-1}\left(\dfrac{1}{32}\right) = \tan^{-1} 9 - \left(\dfrac{\pi}{4}\right)$.

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Provincial Department of Education - NWP

තෙවන වාර පරීක්ෂණය - 12 ශ්‍රේණිය - 2024
Third Term Test - Grade 12 - 2024

Index No.: ....

Combined Mathematics - II

03 hours

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மூன்று மணித்தியாலம்
Three hours

අමතර කියවීම් කාලය - මිනිත්තු 10 යි
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Part	Question No.	Marks
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Total

In numbers	
In words	

PART A

1) A particle is projected from a point O on the level ground inclined to the horizontal at a velocity u.After a time t.,the vertical and the horizontal displacements of the particle are a each.Find u^2 in terms of t, a and g. If $t = 1$, show that $u^2 = 2a^2 + ga + \frac{1}{4}g^2$.

2) In a mine, the elevator starts to move from rest and going down, The first part of the trip moves with uniform acceleration a for a time period of $2T$. The remaining part completes with uniform retardation for a time period of T and comes to rest. If it moves with a distance h,draw a velocity – time graph for the motion and hence ,show that $h = 3aT^2$.

3) The position vectors of two points A and B with respect to a fixed origin O are $\underline{\boldsymbol{a}}$ and $\underline{b}$ respectively, $\underline{a} = \underline{i} + 2\underline{j}$ and $|\underline{b}| = 2$ .The angle between $\underline{\boldsymbol{a}}$ and $\underline{\boldsymbol{b}}$ is $cos^{-1}\left(\frac{1}{\sqrt{10}}\right)$ .Express $\underline{\boldsymbol{b}}$ in the form $\boldsymbol{x}\underline{i} + y\underline{j}$ Where x and y are constants to be determined and $y > 0$.

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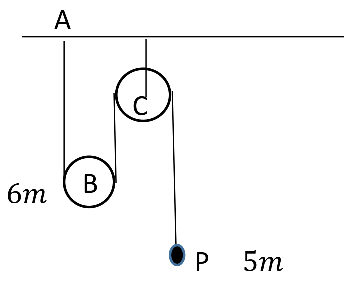
4) One end of a light inextensible string is attached to a fixed point A and the other end is attached to a particle P of mass $5m$, The string goes down a movable pulley B of mass $6m$ and passing over a fixed pulley C as shown in the figure. When the system is released from rest, find the acceleration of the particle P and the tension in the string.



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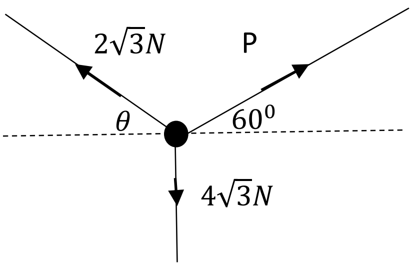
5) Let OAB be a triangle and let $\overrightarrow{OA} = \underline{a}$ and $\overrightarrow{OB} = \underline{b}$ .Suppose that C, D and E are the points with $\overrightarrow{OC} = \underline{a} + \underline{b}$,$\overline{OD} = \frac{1}{2}\underline{a} + \underline{b}$ and $\overrightarrow{OE} = \frac{1}{3}\underline{b}$ respectively. Further if F is the midpoint of OD, show that the points C,F and E are collinear.

6) A ladder of weight W is placed so that one end touching a rough vertical wall and the other end is in contact with a rough horizontal floor, The coefficient of friction of the floor and the wall are μ_1 and μ_2 respectively. Show that the inclination of the ladder to the horizontal when it is in limiting equillibrium is $tan^{-1}\left(\frac{1-\mu_1\mu_2}{2\mu_2}\right)$

7) A motor car A travels at a speed $u\ ms^{-1}$ in the direction θ north of east and another car B moves with a speed $v\ ms^{-1}$ due north. If $v > u\,cosec\,\theta$,by drawing a velocity triangle ,find the magnitude and direction of the speed of A relative to B.

8)



A system of three coplanar forces acting on a particle is shown in the figure. If the particle is in equllibrium , find P and θ.

9) With respect to an Oxy system, the forces $5\underline{i} + \underline{j}$, $-9\underline{i} + \underline{j}$ and $4\underline{i} - 2\underline{j}$ acting at the points with position vectors $3\underline{i} + \underline{j}$, $3\underline{i}$ and $-\underline{i} - 3\underline{j}$ respectively. Show that the system is equivalent to a couple and find the moment and sense of that couple.

10) A particle moving in a straight path in uniform acceleration in the 10^{th} and 16^{th} seconds shows the displacements $29m$ and $41m$ respectively..Show that the displacement in the 25^{th} second is $59m$ using the kinematic equations (Equations of motion)

PART B

❖ **Answer 05 questions only**

11) In a motor car race ,in a certain moment ,the motor car X is about to finish the race and it has to go only a $1100m$ distance ,then its velocity is $38.5ms^{-1}$ and the uniform acceleration is $0.44ms^{-2}$. In that instance ,the motor car Y is $220m$ behind the motor car X and then its velocity is $48.4ms^{-1}$ and the uniform acceleration is $0.55ms^{-2}$. From the above mentioned moment of the race ,sketch velocity – time graphs of the motions of X and Y in the same diagram,

Using the graph,

(i) If the time taken by the motor car Y from the above mentioned moment of the race to overtake motor car X is T , show that T satisfies the equation $T^2 + 180T - 4000 = 0$

(ii) Find the velocities of X and Y, when Y overtakes X

(iii) Show that Y overtakes X at a distance 242m before the finishing line..

(iv) Using equations of motion and show that the time difference of finishing the race for these motor cars is one second,

12) . OACB is a parallelogram. L and M are the mid points of the sides AC and BC respectively. The lines OL and AM intersect at X, Here $\overrightarrow{OA} = \underline{a}$ and $\overrightarrow{OB} = \underline{b}$

(i) Show that $\overrightarrow{OX} = \lambda\left(\underline{a} + \frac{1}{2}\underline{b}\right)$, where $\lambda \in R$

(ii) Obtain another similar expression for $\overrightarrow{OX}$

(iii) Hence, show that $\overrightarrow{OX} = \frac{4}{5}\left(\underline{a} + \frac{1}{2}\underline{b}\right)$

(iv) Find $AX\colon XM$

The lines CX and OA meet at N .**Using the results on equiangular triangles or otherwise**, show that $\overrightarrow{ON} = \frac{2}{3}\underline{a}$

Now the side CA of parallelogram OACB is produced to D such that $CD\colon DA = 3\colon 1$. If the lines OC and BD are perpendicular, show that $|\underline{a}|^2 - \frac{1}{2}\underline{a}.\underline{b} - \frac{3}{2}|\underline{b}|^2 = 0$

If $|\underline{b}| = \beta|\underline{a}|$, where $\beta > 0$ and $A\hat{O}B = tan^{-1}\frac{3}{4}$, show that $\beta = \frac{-2+\sqrt{154}}{15}$

13) . A ship A sails in the direction 60^0 east of north at a velocity $16kmh^{-1}$.At a certain instance,8 Km east of the ship ,a boat B moves at a velocity 10 kmh^{-1}intending to intercept the ship..

(i) Sketch in the same diagram, the velocity triangles to determine the two possible paths of the boat relative to the earth.

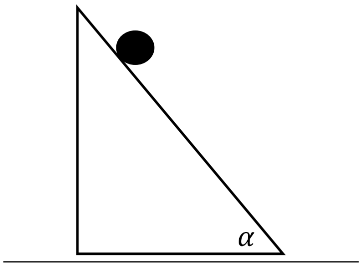
(ii) Show that the angle between those two paths is given by $cos^{-1}\left(\frac{7}{25}\right)$

(iii) Show that the minimum time taken by the boat to intercept the ship is given by $\frac{4}{3+4\sqrt{3}}$ $hours$

(iv) What is the time difference of two paths?

(v) What should be the minimum velocity of the boat to intercept the ship? Explain your answer.

14) As shown in the figure, a smooth wedge of mass M is placed on a smooth horizontal plane. A particle of mass m is kept on its face with inclination α to the horizontal and released the system from rest.



(i) Show that the acceleration of the wedge is given by

$\frac{mgsin\alpha cos\alpha}{M+msin^2\alpha}$

(ii) When the particle moves a distance S along the surface of the wedge ,the wedge moves a distance d along the horizontal plane .Show that $\left(1+\frac{M}{m}\right)d = Scos\alpha$

(iii) Show that the reaction between the wedge and the particle is $\frac{Mmgcos\alpha}{M+msin^2\alpha}$

(iv) Show also that the reaction between the horizontal palne and the wedge is $\frac{M(M+m)g}{M+msin^2\alpha}$

15) A ball is projected so that it passes over two parallel walls. The velocity of projection is u and the angle of projection is θ. The two walls are at distances of b and a from the point of projection and are of heights a and b respectively.

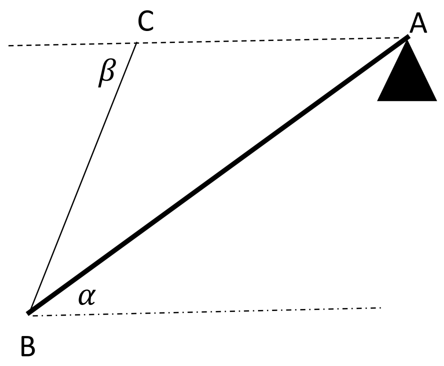
(i) Show that $a = btan\theta - \frac{gb^2}{2u^2}sec^2\theta$

(ii) Show also that $tan\theta = \frac{a^2+ab+b^2}{ab}$

(iii) show that the distance to the second wall from the point where the ball reaches the ground is $\frac{b^2}{a+b}$

(iv) Show also that $\theta > tan^{-1}3$

16) As shown in the figure, the upper end A of a uniform rod AB of length a and weight W,which is in equillibrium inclined at an angle α to the horizontal, touches a smooth peg and a string attached to a point C on the same level as A is attached to its lower end B.The inclination of the string to the horizontal is β.



(i) Mark the forces in the figure.

(ii) Find the tension in the string

(iii) Find the reaction of the peg on the rod

(iii) Show that $tan\beta = 2tan\alpha + cot\alpha$

(iv) Show also that $AC = \frac{asec\alpha}{2tan^2\alpha+1}$

(v) If $\alpha = \frac{\beta}{2}$,using the above results ,show that

$\beta = 2tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$ and $AC = \frac{\sqrt{6}a}{4}$

17) (a) ABCDEF is a regular hexagon of length of a side a.The forces of magnitudes P , $4P$, $2P$, $2P$,$3P$ and $3P$ act along $\overrightarrow{AB}$,$\overrightarrow{BC}$, $\overrightarrow{CD}$,$\overrightarrow{DE}$,$\overrightarrow{EF}$ and $\overrightarrow{FA}$ respectively.

(i) show that the system of forces is equivalent to a couple and find the magnitude and sense of the couple.

(ii) Now the force $3P$ along $\overrightarrow{FA}$ is removed from the system. **Hence or otherwise,** find the magnitude direction of the new system and also the distance to the point where the resultant force cuts the line AB produced from A.

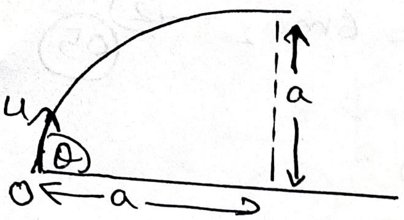
(iii) Now a clockwise couple of magnitude $\frac{15Pa}{2}$ is added to the above new system and the system is reduced to a single force acting through the midpoint of AB with a couple. Find the magnitude and direction of that single force and the magnitude and sense of the couple.

(b) The end A of a uniform rod AB of length $2a$ and weight W is in contact with a rough vertical wall and the other end of the light inextensible string tied to the end B is connected to the fixed point C on the wall which is vertically above A. The rod lies in the vertical plane perpendicular to the wall. If $A\hat{C}B = 30^0$ and $C\hat{A}B = 120^0$, show that the minimum value of the coefficient of friction between the wall and the rod is $\frac{1}{\sqrt{3}}$.

Marking Scheme
Third term test – 2024

Pg – 01

(01)



$\overrightarrow{S} = ut$

$a = u\cos\theta\, t$

$\cos\theta = \dfrac{a}{ut}$ (05)

$S = ut + \frac{1}{2}at^2 \uparrow$

$a = u\sin\theta\, t - \frac{1}{2}gt^2$

$\sin\theta = \dfrac{a}{ut} + \dfrac{gt}{2u}$ (05)

Since $\sin^2\theta + \cos^2\theta = 1$ (05)

$$\frac{a^2}{u^2t^2} + \frac{1}{u^2}\left[\frac{a}{t} + \frac{gt}{2}\right]^2 = 1$$

$$\frac{a^2}{t^2} + \left[\frac{a}{t} + \frac{gt}{2}\right]^2 = u^2 \quad (05)$$

If $t = 1$

$$a^2 + a^2 + ag + \frac{g^2}{4} = u^2$$

$$2a^2 + ag + \frac{g^2}{4} = u^2 \quad (05)$$

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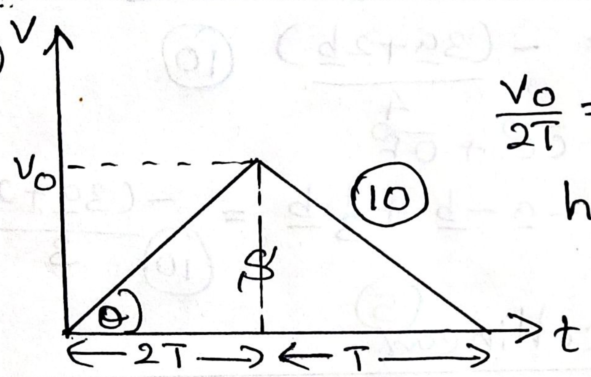
(02)



$\tan\theta = a$, $S = h$

$\dfrac{V_0}{2T} = a \Rightarrow V_0 = 2aT$ (05)

$h = \frac{1}{\cancel{2}} \times 3T \times \cancel{2}aT$ (05)

$= 3aT^2$ (05)

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(03) $\underline{a} \cdot \underline{b} = |\underline{a}||\underline{b}|\cos\theta$

$(\underline{i} + 2\underline{j}) \cdot (x\underline{i} + y\underline{j}) = \sqrt{5} \times 2 \times \frac{1}{\sqrt{10}}$ (05)

$x + 2y = \sqrt{2}$ — ① (05)

$\sqrt{x^2 + y^2} = 2$

$x^2 + y^2 = 4$ — ② (05)

$(\sqrt{2} - 2y)^2 + y^2 = 4$

$5y^2 - 4\sqrt{2}y - 2 = 0$

$(5y + \sqrt{2})(y - \sqrt{2}) = 0$

$y = -\frac{\sqrt{2}}{5}$ or $y = \sqrt{2}$

since $y > 0$,

$y = \sqrt{2}$ (05)

$\therefore x = -\sqrt{2}$

$\therefore \underline{b} = -\sqrt{2}\underline{i} + \sqrt{2}\underline{j}$ // (05)

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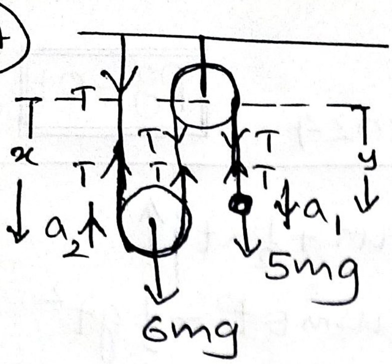
(04)



(05)

5m $F = ma\downarrow$

$5mg - T = 5ma_1$ —— (1) (05)

6m $F = ma\uparrow$

$2T - 6mg = 6ma_2$ —— (2) (05)

$2x + y = l$

$2\ddot{x} + \ddot{y} = 0$

$2(-a_2) + a_1 = 0$

$a_1 = 2a_2$ —— (3)

(1)×2 + (2) (05)

$4g = 10a_1 + 6a_2$

$a_2 = \frac{2g}{13}$, $a_1 = \frac{4g}{13}$, $T = \frac{45mg}{13}$ (05)

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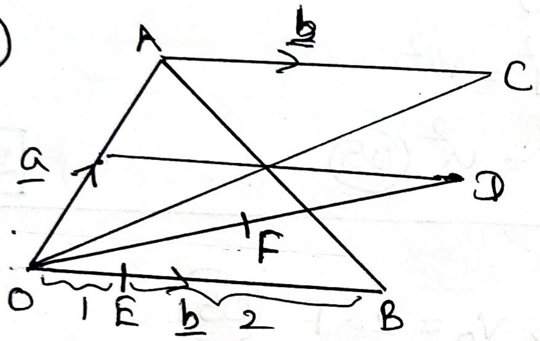
(05)



$\overrightarrow{CF} = \overrightarrow{CO} + \overrightarrow{OF}$

$= -\underline{a} - \underline{b} + \frac{1}{2}\overrightarrow{OD}$

$= -\underline{a} - \underline{b} + \frac{1}{2}\left[\frac{1}{2}\underline{a} + \underline{b}\right]$

$= -\frac{(3\underline{a} + 2\underline{b})}{4}$ (10)

$\overrightarrow{CE} = \overrightarrow{CO} + \overrightarrow{OE}$

$= -\underline{a} - \underline{b} + \frac{1}{3}\underline{b} = -\frac{(3\underline{a} + 2\underline{b})}{3}$ (10)

$\therefore 4\overrightarrow{CF} = 3\overrightarrow{CE}$

$\therefore$ C, F, E are collinear. (5)

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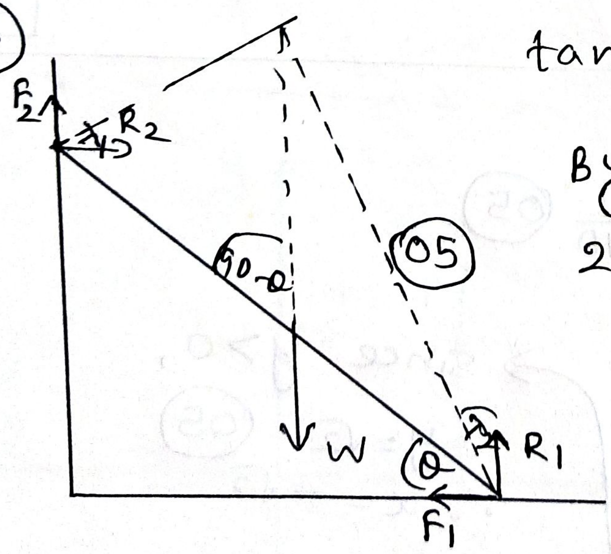
(06)



(05)

$\tan\lambda_1 = \mu_1 \quad \tan\lambda_2 = \mu_2$

By cot theorem

$2\cot(90 - \theta) = \cot\lambda_2 - \cot(90 - \lambda_1)$ (10)

$2\tan\theta = \frac{1}{\tan\lambda_2} - \tan\lambda_1$ (05)

$2\tan\theta = \frac{1}{\mu_2} - \mu_1$

$\tan\theta = \frac{1 - \mu_1\mu_2}{2\mu_2}$ (05)

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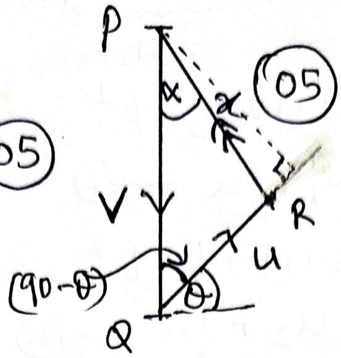
(07) $V_{A,E} = \overset{u}{\nearrow}\ (\theta)$

$V_{B,E} = \uparrow V$

$V_{A,B} = V_{A,E} + V_{E,B}$ (05)

$PR = \underset{Q}{\overset{u \ R}{\nearrow}}\ (\theta) + \underset{Q}{\overset{P}{\downarrow}}\ v$ (05)



By cosine rule

$x^2 = u^2+v^2-2uv\cos(90-\theta)$

$x = \sqrt{u^2+v^2-2uv\sin\theta}$ (05)

By sine rule

$\frac{u}{\sin\alpha} = \frac{x}{\sin(90-\theta)}$

$\sin\alpha = \frac{u\cos\theta}{\sqrt{u^2+v^2-2uv\sin\theta}}$

$\therefore \alpha = \sin^{-1}\left[\frac{u\cos\theta}{\sqrt{u^2+v^2-2uv\sin\theta}}\right]$ (05)

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(08) $\rightarrow$ $P\cos 60^\circ = 2\sqrt{3}\cos\theta$ — ① (05)

$\uparrow$ $P\sin 60^\circ + 2\sqrt{3}\sin\theta = 4\sqrt{3}$ — ② (05)

By ① $P = 4\sqrt{3}\cos\theta$

By ② $\sin\theta = \frac{8-P}{4}$

Since $\sin^2\theta + \cos^2\theta = 1$

$\left(\frac{8-P}{4}\right)^2 + \left(\frac{P}{4\sqrt{3}}\right)^2 = 1$ (05)

$P = 6$ (05)

$\therefore \cos\theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = \pi/6$ (05)

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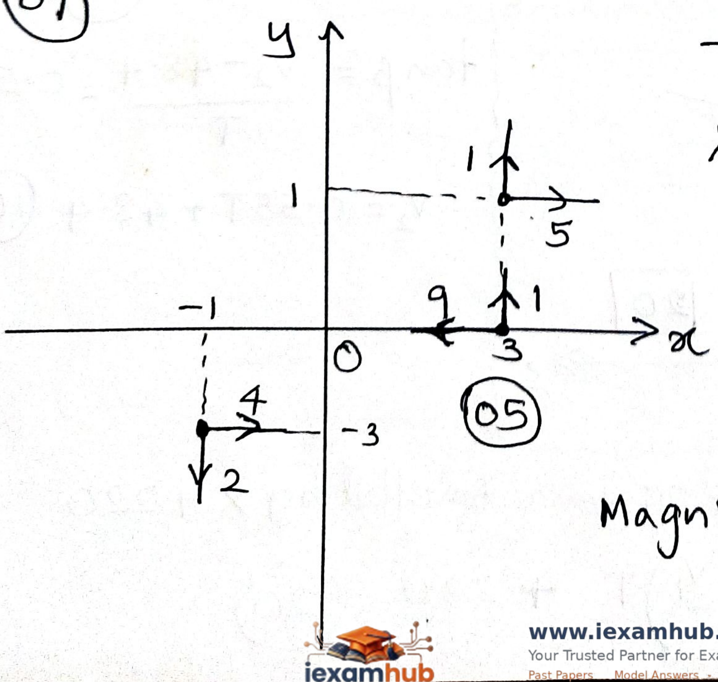
(09)



$\vec{X} = 4+5-9 = 0$ (05)

$\uparrow Y = 1+1-2 = 0$ (05)

$\overset{\curvearrowleft}{G_0} = 1\times1 - 5\times1 + 1\times3 + 4\times3 + 2\times1$

$= 1-5+3+12+2$

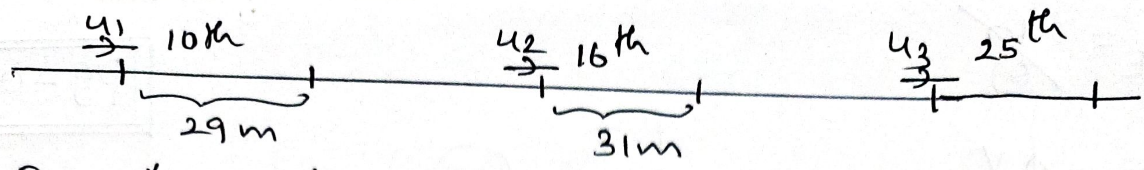
$= 13$ Nm. (05)

Magnitude of the couple = 13 Nm

Sense of the couple = anticlockwise (05)

25

(10)



For 10^{th} second, $s = ut + \frac{1}{2}at^2$

$29 = u_1 + \frac{1}{2}a$ — (1)

For 16^{th} second, $s = ut + \frac{1}{2}at^2$

$41 = u_2 + \frac{1}{2}a$ — (2) (05)

(2) − (1) $u_2 - u_1 = 12$ — (3)

applying $v = u + at$

$u_2 = u_1 + 6a$

$u_2 - u_1 = 6a$ — (4) (05)

By (3) & (4) $a = 2\,ms^{-2}$ (05), $u_2 = 40\,ms^{-1}$

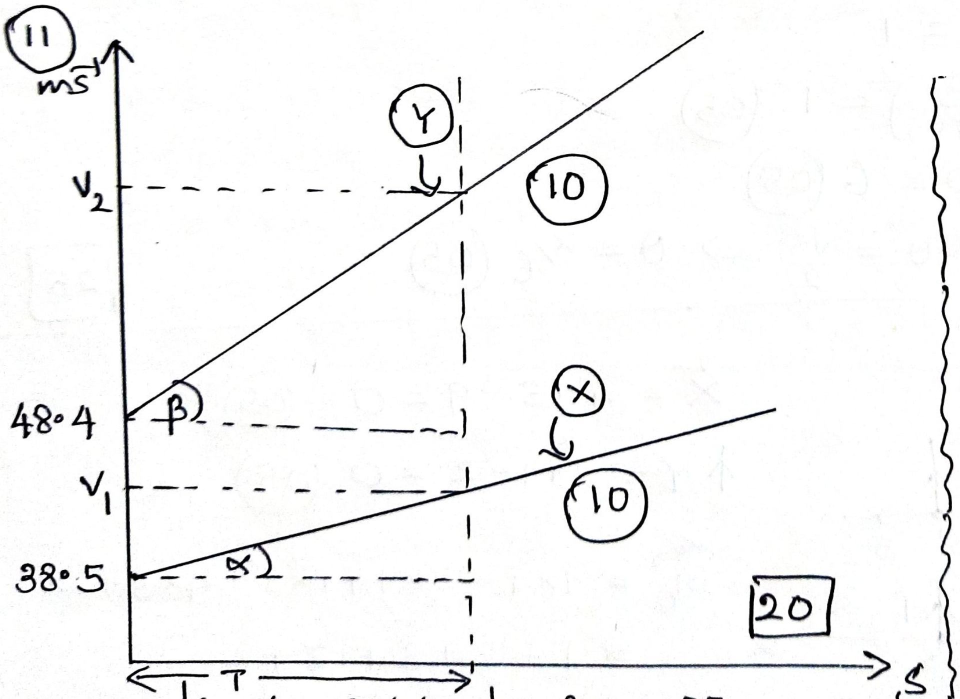
$v = u + at$ $u_3 = 40 + 2 \times 9 = 58\,ms^{-1}$ (05)

For 25^{th} second, $s = ut + \frac{1}{2}at^2$

$= 58 + \frac{1}{2} \times 2 = 59\,m$ (05) $\boxed{25}$

PART B

(11)



$\tan\alpha = 0.44$, $\tan\beta = 0.55$

$\tan\alpha = \frac{V_1 - 38.5}{T} = 0.44$

$V_1 = 0.44T + 38.5$ (10)

$\tan\beta = \frac{V_2 - 48.4}{T} = 0.55$

$V_2 = 0.55T + 48.4$ (10)

Distance travelled by Y = Distance travelled by X + 220

$\left(\frac{48.4 + V_2}{2}\right)T = \left(\frac{38.5 + V_1}{2}\right)T + 220$ (20)

$96.8T + 0.55T^2 = 77T + 0.44T^2 + 440$

$0.11T^2 + 19.8T - 440 = 0$

$T^2 + 180T - 4000 = 0$ (30) $\boxed{70}$

(ii) $(T+200)(T-20) = 0$

Since $T > 0$, $T = 20s$. (10)

$V_1 = 8.8 + 38.5$
$= 47.3 ms^{-1}$ (10)

$V_2 = 11.00 + 48.4$
$= 59.4 ms^{-1}$ (10)

$\boxed{30}$

(iii) Distance travelled by $x = \left(\frac{38.5 + 47.3}{2}\right) 20$

$= 858m$ (10)

Distance to the finishing line $= 1100 - 858 = 242m$ $\boxed{10}$

(iv) For x, $\vec{S = ut + \frac{1}{2}at^2}$

$242 = 47.3t + \frac{1}{2} \times 0.44t^2$

$t^2 + 215t - 1100 = 0$

$(t+220)(t-5) = 0$

Since $t > 0$, $t = 5s$ (10)

For Y, $S = ut + \frac{1}{2}at^2$

$242 = 59.4t + \frac{1}{2} \times 0.55t^2$

$t^2 + 216t - 880 = 0$

$(t+220)(t-4) = 0$

Since $t > 0$, $t = 4s$ (10)

$\therefore$ time difference $= 1s$ $\boxed{20}$

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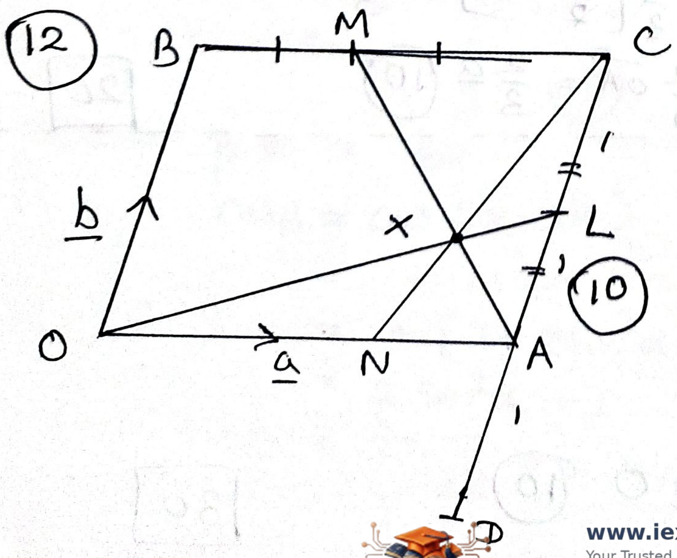
(12)



(10)

(i) $\vec{OX} = \lambda \vec{OL}$

$= \lambda[\vec{OA} + \vec{AL}]$

$= \lambda[\underline{a} + \frac{1}{2}\vec{AC}]$

$= \lambda[\underline{a} + \frac{1}{2}\underline{b}]$ (10)

$\boxed{20}$

(ii) $\overrightarrow{OX} = \overrightarrow{OA} + \overrightarrow{AX}$

$= \underline{a} + \mu \overrightarrow{AM}$

$= \underline{a} + \mu [\overrightarrow{AC} + \overrightarrow{CM}]$

$= \underline{a} + \mu [\underline{b} - \frac{1}{2}\underline{a}]$ (10)

$\boxed{10}$

(iii) $\lambda [\underline{a} + \frac{1}{2}\underline{b}] = \underline{a} + \mu [\underline{b} - \frac{1}{2}\underline{a}]$

$\underline{a}[\lambda - 1 + \frac{\mu}{2}] + \underline{b}[\frac{\lambda}{2} - \mu] = \underline{0}$ (10)

Since $\underline{a} \not\parallel \underline{b}$

$\frac{\lambda}{2} = \mu$

$\lambda = 2\mu$ — (1) (5)

$\lambda - 1 + \frac{\mu}{2} = 0$ (5)

$5\lambda = 4$

$\lambda = \frac{4}{5}$ (5), $\mu = \frac{2}{5}$

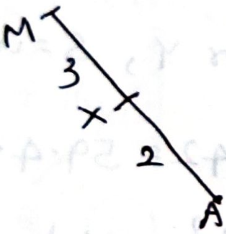
$\therefore \overrightarrow{OX} = \frac{4}{5}(\underline{a} + \frac{1}{2}\underline{b})$ (5)

$\boxed{30}$

(iv) $\overrightarrow{AX} = \mu \overrightarrow{AM}$

$AX = \frac{2}{5} AM$

$\frac{AX}{AM} = \frac{2}{5}$



$\therefore AX : XM = 2 : 3$ (10)

$\boxed{10}$

Since $MXC\Delta$ and $NXA\Delta$ are equiangular

$\frac{NA}{MC} = \frac{AX}{XM}$ (10)

$\frac{NA}{MC} = \frac{2}{3} \Rightarrow \overrightarrow{NA} = \frac{2}{3}\overrightarrow{MC}$

$\overrightarrow{NA} = \frac{\cancel{2}}{3}[\frac{1}{\cancel{2}}\overrightarrow{OA}] = \frac{1}{3}\overrightarrow{OA}$

$\therefore \overrightarrow{ON} = \frac{2}{3}\overrightarrow{OA} = \frac{2}{3}\underline{a}$ (10)

$\boxed{20}$

Since $\overrightarrow{OC} \cdot \overrightarrow{BD} = 0$

$(\underline{a} + \underline{b}) \cdot (\overrightarrow{BO} + \overrightarrow{OD}) = 0$ (10)

$(\underline{a} + \underline{b}) \cdot (-\underline{b} + \underline{a} - \frac{\underline{b}}{2}) = 0$

$(\underline{a} + \underline{b}) \cdot (\underline{a} - \frac{3}{2}\underline{b}) = 0$ (10)

$|\underline{a}|^2 - \frac{1}{2}\underline{a} \cdot \underline{b} - \frac{3}{2}|\underline{b}|^2 = 0$ (10)

$\boxed{30}$

$$|\underline{a}|^2 - \frac{1}{2}|\underline{a}||\underline{b}| \times \frac{4}{5} - \frac{3}{2}\beta^2|\underline{a}|^2 = 0 \quad (10)$$

$$\cancel{|\underline{a}|^2} - \frac{2}{5}\beta\cancel{|\underline{a}|^2} - \frac{3}{2}\beta^2\cancel{|\underline{a}|^2} = 0$$

$$1 - \frac{2}{5}\beta - \frac{3}{2}\beta^2 = 0$$

$$15\beta^2 + 4\beta - 10 = 0 \quad (10)$$

$$\beta = \frac{-4 \pm \sqrt{16+600}}{30} = \frac{-4 \pm 2\sqrt{154}}{30}$$

Since $\beta > 0$, $\beta = \dfrac{-2+\sqrt{154}}{15}$ (10)

$\boxed{30}$

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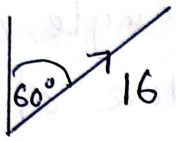
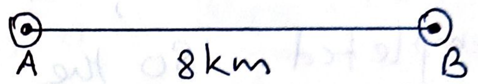
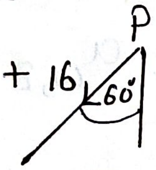
(13) $V_{A,E} =$  (5)



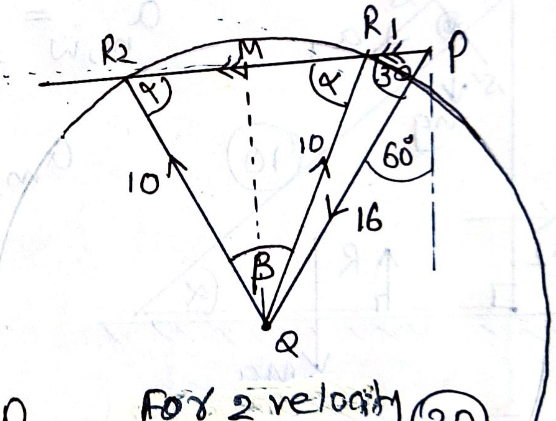
$V_{B,E} = 10$ (5)

(i) $V_{B,A} = V_{B,E} + V_{E,A}$ (5)

$\overleftarrow{RP} = 10 + $ 

(5) (5) (5)

there are 2 directions, $\overrightarrow{QR_1}$ and $\overrightarrow{QR_2}$, that the boat can move relative to the earth, to intercept ship.



For 2 velocity triangles (30)

$\boxed{60}$

(ii) $QM = 10\sin\alpha = 16\sin 30^\circ$

$\sin\alpha = \frac{4}{5}$ (10)

$$\beta = \pi - 2\alpha$$

$$\cos\beta = \cos(\pi - 2\alpha)$$
$$= -\cos 2\alpha$$
$$= -[1 - 2\sin^2\alpha]$$
$$= 2\sin^2\alpha - 1$$

$$\cos\beta = 2 \times \frac{16}{25} - 1 = \frac{7}{25} \quad (15)$$

$\therefore \beta = \cos^{-1}\left(\frac{7}{25}\right)$ (5)

$\boxed{30}$

(iii) minimum time $(t_1) = \frac{8}{PR_2}$ (5) $= \frac{8}{16\cos 30^\circ + 10\cos\alpha}$ (10)

$= \frac{8}{8\sqrt{3}+6} = \frac{4}{3+4\sqrt{3}}$ (5) $\boxed{20}$

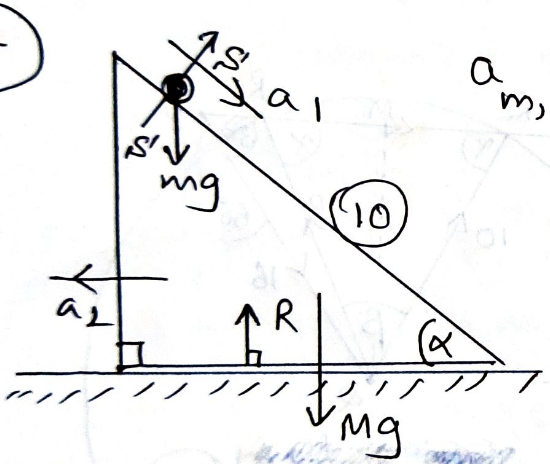
(iv) $t_2 = \frac{8}{PR_1}$ (5) $= \frac{8}{16\cos 30^\circ - 10\cos\alpha}$ (5) $= \frac{8}{8\sqrt{3}-6} = \frac{4}{4\sqrt{3}-3}$

$t_2 - t_1 = 4\left[\frac{\cancel{4\sqrt{3}}+3-\cancel{4\sqrt{3}}+3}{48-9}\right] = \frac{24}{39} = \frac{8}{13}\, s$ (10) (10) $\boxed{30}$

(v) If $QM < 16\sin 30^\circ$, a velocity triangle cannot be completed. So the minimum value of the speed of the boat to intercept the ship is

$16\sin 30^\circ = 8\, ms^{-1}$. $\boxed{10}$

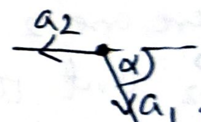
(14)



$a_{m,w} = \searrow a_1$ (α) $\quad a_{w,E} = \xleftarrow{} a_2$

$a_{m,E} = a_{m,w} + a_{w,E}$ (10)

$= \searrow a_1 \,(\alpha) + \xleftarrow{} a_2 =$



$\boxed{20}$

For the particle, $F = ma \searrow$

$mg\sin\alpha = m(a_1 - a_2\cos\alpha)$

$g\sin\alpha = a_1 - a_2\cos\alpha$ — (1) (10)

For the system, $\overleftarrow{F = ma}$

$0 = Ma_2 + m[a_2 - a_1\cos\alpha]$ — (2) (20) $\boxed{30}$

(i) By (1) $\Rightarrow a_1 = g\sin\alpha + a_2\cos\alpha$

By (2) $\Rightarrow 0 = (M+m)a_2 - ma_1\cos\alpha$

$0 = (M+m)a_2 - m\cos\alpha\,(g\sin\alpha + a_2\cos\alpha)$

$$a_2[M+m-m\cos^2\alpha] = mg\sin\alpha\cos\alpha$$

$$a_2 = \frac{mg\sin\alpha\cos\alpha}{M+m\sin^2\alpha} \quad (20)$$

$\boxed{20}$

(ii) For the particle

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}a_1 t_0^2$

$t_0^2 = \frac{2s}{a_1} \quad (10)$

For the wedge

$s = ut + \frac{1}{2}at^2$

$d = \frac{1}{2}a_2 t_0^2$

$d = \frac{1}{\cancel{2}} a_2 \times \frac{\cancel{2}s}{a_1}$

$\frac{d}{s} = \frac{a_2}{a_1} \quad (10)$

$$a_1 = g\sin\alpha + \frac{mg\sin\alpha\cos^2\alpha}{M+m\sin^2\alpha} \Rightarrow a_1 = \frac{(M+m)g\sin\alpha}{M+m\sin^2\alpha} \quad (10) \text{ //}$$

$$\therefore \frac{d}{s} = \frac{m\cancel{g}\cancel{\sin\alpha}\cos\alpha}{(M+m)\cancel{g}\cancel{\sin\alpha}}$$

$$d(M+m) = ms\cos\alpha$$

$$d\left[\frac{M}{m}+1\right] = s\cos\alpha \quad (10)$$

$\boxed{40}$

(iii) For the particle $F = ma$

$$S - mg\cos\alpha = m(-a_2\sin\alpha) \quad (10)$$

$$S = mg\cos\alpha - m\sin\alpha\left[\frac{mg\sin\alpha\cos\alpha}{M+m\sin^2\alpha}\right]$$

$$= mg\cos\alpha\left[\frac{M+\cancel{m\sin^2\alpha}-\cancel{m\sin^2\alpha}}{M+m\sin^2\alpha}\right]$$

$$= \frac{Mmg\cos\alpha}{M+m\sin^2\alpha} \quad (10)$$

$\boxed{20}$

(iv) For the system $F = ma$

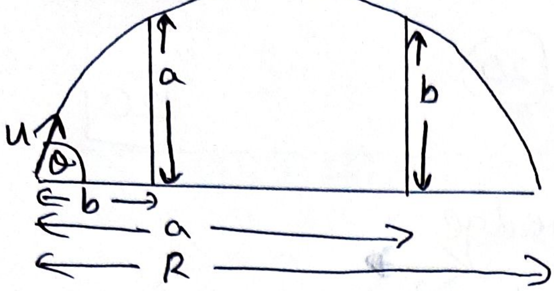
$$R - Mg - mg = m(-a_1\sin\alpha) \quad (10)$$

$$R = (M+m)g - \frac{m(M+m)g\sin^2\alpha}{M+m\sin^2\alpha}$$

$$= (M+m)g\left[\frac{M+\cancel{m\sin^2\alpha}-\cancel{m\sin^2\alpha}}{M+m\sin^2\alpha}\right] = \frac{M(M+m)g}{M+m\sin^2\alpha} \quad (10)$$

$\boxed{20}$

15



(i) $S = \overrightarrow{u}t$

$b = u\cos\theta\, t_1 \Rightarrow t_1 = \dfrac{b}{u\cos\theta}$ 10

$S = ut + \frac{1}{2}at^2 \uparrow$ 10

$\Rightarrow a = \cancel{u}\sin\theta \times \dfrac{b}{\cancel{u}\cos\theta} - \dfrac{1}{2}g\dfrac{b^2}{u^2\cos^2\theta}$

$a = b\tan\theta - \dfrac{gb^2}{2u^2}\sec^2\theta \rightsquigarrow (1)$ 10 $\boxed{30}$

(ii) $S = \overrightarrow{u}t$

$a = u\cos\theta\, t_2$

$t_2 = \dfrac{a}{u\cos\theta}$ 5

$S = ut + \frac{1}{2}at^2 \uparrow$

$b = u\sin\theta \times \dfrac{a}{u\cos\theta} - \dfrac{1}{2}g\dfrac{a^2}{u^2\cos^2\theta}$ 5

$b = a\tan\theta - \dfrac{ga^2}{2u^2}\sec^2\theta \rightsquigarrow (2)$ 5

by (1) $\Rightarrow \dfrac{(1)}{b^2} \Rightarrow \dfrac{a}{b^2} = \dfrac{1}{b}\tan\theta - \dfrac{g}{2u^2}\sec^2\theta \rightsquigarrow (3)$ 5

$\dfrac{(2)}{a^2} \Rightarrow \dfrac{b}{a^2} = \dfrac{1}{a}\tan\theta - \dfrac{g}{2u^2}\sec^2\theta \rightsquigarrow (4)$ 5

$(3) - (4) \quad \dfrac{a}{b^2} - \dfrac{b}{a^2} = \left(\dfrac{1}{b} - \dfrac{1}{a}\right)\tan\theta$ 5

$\dfrac{a^3 - b^3}{a^2b^2} = \left(\dfrac{a-b}{ab}\right)\tan\theta$

$\dfrac{\cancel{(a-b)}(a^2+ab+b^2)}{a^{\cancel{2}}b^{\cancel{2}}} = \left(\dfrac{\cancel{a-b}}{\cancel{ab}}\right)\tan\theta$

$\tan\theta = \dfrac{a^2+ab+b^2}{ab}$ 10 $\boxed{40}$

(iii) By (1), we substitute $b = R$ and $a = 0$

$0 = R\tan\theta - \dfrac{gR^2}{2u^2}\sec^2\theta$

$\dfrac{gR}{2u^2}\sec^2\theta = \tan\theta$

$R = \dfrac{2u^2\tan\theta}{g\sec^2\theta} = \dfrac{2u^2\sin\theta\cos\theta}{g}$ 20

By ① $\Rightarrow$ $a = b\tan\theta - \frac{gb^2}{2u^2}\sec^2\theta$

$$\frac{gb^2}{2u^2}\sec^2\theta = b\tan\theta - a$$

$$u^2 = \frac{gb^2\sec^2\theta}{2(b\tan\theta - a)} \quad 10$$

$$\therefore R = \frac{\cancel{2}\sin\theta\cos\theta}{\cancel{g}} \times \frac{\cancel{g}b^2\sec^2\theta}{\cancel{2}(b\tan\theta - a)} = \frac{b^2\tan\theta}{(b\tan\theta - a)}$$

$$R = \frac{b^{\cancel{2}}\left(\frac{a^2+b^2+ab}{a\cancel{b}}\right)}{\cancel{b}\left(\frac{a^2+b^2+ab}{a\cancel{b}}\right) - a} = \frac{b[a^2+b^2+ab]}{b^2+ab} = \frac{a^2+b^2+ab}{b+a} \quad 20$$

Distance from second wall $= R - a$

$$= \frac{a^2+b^2+ab}{a+b} - a \quad 10$$

$$= \frac{b^2}{a+b}$$

$\boxed{60}$

(iv) Since $(a-b)^2 \geq 0$

$$a^2+b^2-2ab \geq 0$$

$$a^2+b^2 \geq 2ab$$

$$a^2+b^2+ab \geq 3ab$$

$$\frac{a^2+b^2+ab}{ab} \geq 3$$

$$\tan\theta \geq 3$$

$$\theta \geq \tan^{-1}(3)$$

$\boxed{20}$

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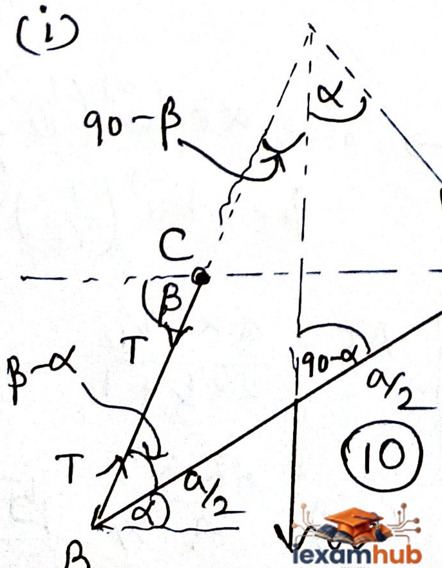
16 (i)



$\boxed{10}$

(ii) Taking moments about A, for the rod.

$$T\sin(\beta-\alpha) \times \cancel{a} = W\cos\alpha \times \frac{\cancel{a}}{2}$$

$$T = \frac{W\cos\alpha}{2\sin(\beta-\alpha)}$$

$\boxed{20}$

(iii) Taking moments about B, for the rod

$$R \times a = W\cos\alpha \times a/2$$

$$R = \frac{W\cos\alpha}{2}$$

$\boxed{20}$

(iv) By cot theorem

$$2\cot(90-\alpha) = \cot(90-\beta) - \cot\alpha$$

$$2\tan\alpha = \tan\beta - \cot\alpha$$

$$\tan\beta = 2\tan\alpha + \cot\alpha$$

$\boxed{20}$

(v) By applying sine rule for ΔABC

$$\frac{AC}{\sin(\beta-\alpha)} = \frac{a}{\sin\beta}$$

$$AC = \frac{a\sin(\beta-\alpha)}{\sin\beta} = \frac{a[\sin\beta\cos\alpha - \cos\beta\sin\alpha]}{\sin\beta}$$

$$= a[\cos\alpha - \cot\beta\sin\alpha]$$

But $\tan\beta = \frac{2\sin\alpha}{\cos\alpha} + \frac{\cos\alpha}{\sin\alpha}$

$$= \frac{2\sin^2\alpha + \cos^2\alpha}{\sin\alpha\cos\alpha} = \frac{1+\sin^2\alpha}{\sin\alpha\cos\alpha}$$

$$\therefore AC = a\left[\cos\alpha - \frac{\sin^2\alpha\cos\alpha}{1+\sin^2\alpha}\right] = \frac{a[\cos\alpha]}{1+\sin^2\alpha}$$

$$AC = \frac{a\sec\alpha}{\sec^2\alpha + \tan^2\alpha} = \frac{a\sec\alpha}{2\tan^2\alpha + 1} \;//$$

$\boxed{30}$

(vi) Since $\alpha = \frac{\beta}{2} \Rightarrow \beta = 2\alpha$

Since $\tan\beta = 2\tan\alpha + \cot\alpha$

$$\tan 2\alpha = 2\tan\alpha + \cot\alpha \quad (10)$$

$$\frac{2\tan\alpha}{1-\tan^2\alpha} = 2\tan\alpha + \frac{1}{\tan\alpha}$$

If $t = \tan\alpha \Rightarrow 2t^4 + t^2 - 1 = 0$

(10) $(t^2+1)(2t^2-1) = 0$

$$t = \pm 1/\sqrt{2}$$

Since α is acute, $\tan\alpha = \frac{1}{\sqrt{2}}$

$\therefore \alpha = \tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$

$\beta = 2\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$ (10)

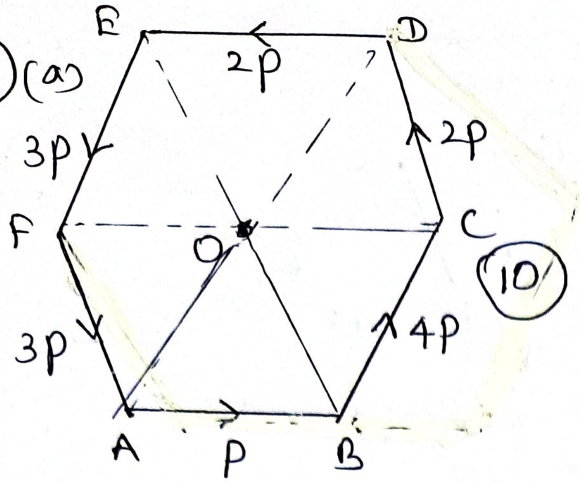
$$AC = \frac{a \times \sqrt{3}/\sqrt{2}}{2 \times \frac{1}{2} + 1}$$

$$= \frac{\sqrt{6}\,a}{4} \;//$$ (20)

$\boxed{50}$

Pg – 07

(17) (a)



(i) $\vec{X} = p + 4p\cos 60^\circ - 2p\cos 60^\circ - 2p - 3p\cos 60^\circ + 3p\cos 60^\circ$

$= p + 2p - p - 2p - \frac{3p}{2} + \frac{3p}{2} = 0$ (10)

$\uparrow Y = 4p\sin 60^\circ + 2p\sin 60^\circ - 3p\sin 60^\circ - 3p\sin 60^\circ$

$= 0$ (10)

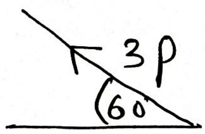
$G_O = a\sin 60^\circ (15p) = \frac{15\sqrt{3}\,pa}{2}$ Nm

The system is equivalent to an anticlockwise couple of magnitude $\frac{15\sqrt{3}\,pa}{2}$. (20)

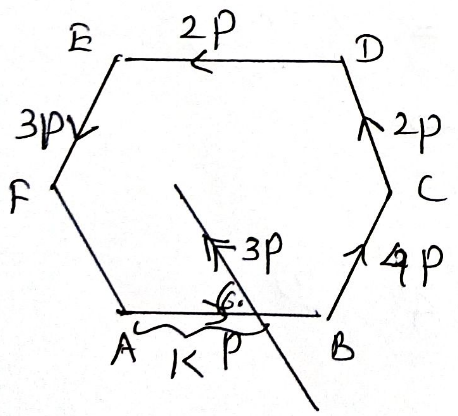
$\boxed{50}$

(ii) The resultant should be same in magnitude and opposite in direction to the removed force.

∴ The resultant force is



(20)



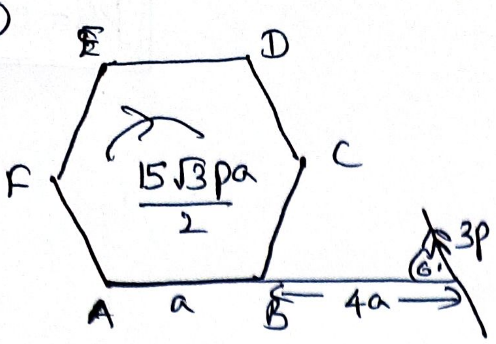
$3p\sin 60^\circ \times k = 4p\sin 60^\circ \times a + 2p\sin 60^\circ \times 2a + 2p\sin 60^\circ \times 2a + 3p\sin 60^\circ \times a$

$3k = 4a + 4a + 4a + 3a$

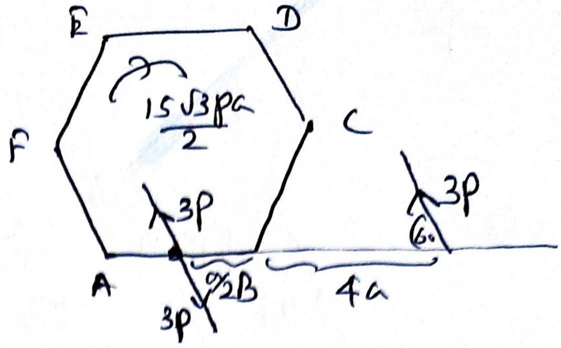
$k = 5a$ (20)

$\boxed{40}$

(iii)



$\equiv$



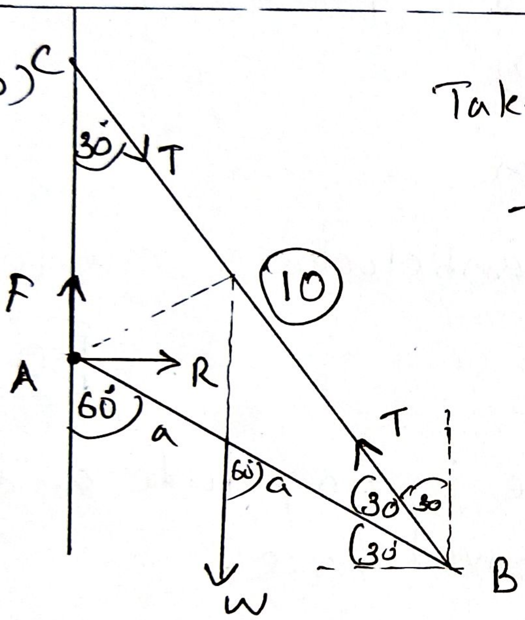
Single force =  (5)

$$\overset{\curvearrowright}{G} = \frac{15\sqrt{3}a}{2} - 3p\sin 60^\circ \times \left(4a + \frac{a}{2}\right)$$

$$= \frac{15\sqrt{3}a}{2} - \frac{27\sqrt{3}pa}{4} = \frac{3\sqrt{3}pa}{4}$$ (15)

$\boxed{20}$

(b)



Taking moments for the rod about A $\curvearrowright$

$$T\sin 30^\circ \times 2a = W\sin 60^\circ \times a$$

$$T = \frac{\sqrt{3}W}{2}$$ (10)

Resolving vertically $\uparrow$

$$F + T\cos 30^\circ = W$$

$$F = W - \frac{\sqrt{3}W}{2} \times \frac{\sqrt{3}}{2}$$

$$F = W - \frac{3W}{4} = \frac{W}{4}$$ (5)

Resolving horizontally $\rightarrow$

$$R = T\cos 60^\circ$$

$$= \frac{\sqrt{3}W}{2} \times \frac{1}{2} = \frac{\sqrt{3}W}{4}$$ (5)

For static equilibrium

$$\frac{F}{R} \le \mu \Rightarrow \frac{W/4}{\sqrt{3}W/4} \le \mu \Rightarrow \frac{1}{\sqrt{3}} \le \mu$$ (10)

minimum value of μ is $\frac{1}{\sqrt{3}}$

$\boxed{40}$