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Provincial Department of Education - NWP

තෙවන වාර පරීක්ෂණය - 13 ශ්‍රේණිය - 2024
Third Term Test - Grade 13 - 2024

විභාග අංකය: ....

Combined Mathematics - I

Time: 03 Hours

පැය තුනයි
மூன்று மணித்தியாலம்
Three hours

අමතර කියවීම් කාලය - මිනිත්තු 10 යි
மேலதிக வாசிப்பு நேரம் - 10 நிமிடங்கள்
Additional Reading Time - 10 minutes

Use additional reading time to go through the question paper, select the questions you will answer decide which of them you will prioritise.

Index number | | | | | | | |

Instructions:

- *This question paper consists of two parts;* **Part A** (Questions 1 - 10) and **Part B** (Questions 11 - 17)
- **Part A:** *Answer **all** questions. Write your answers to each question in the space provided. You may use additional sheets if more space is needed.*
- **Part B:** *Answer **five** questions only. Write your answers on the sheets provided*
- *At the end of the time allotted, tie the answer scripts of the two parts together so that* **Part A** *is on the top of* **Part B** *and hand them over to the supervisor.*
- *You are permitted to remove **only*** **Part B** *of the question paper from the Examination Hall*

For Examiners' Use only

(10) Combined Mathematics I		
Part	Question No.	Marks
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	

Total

In numbers	
In words	

Part A

01. Using the principle of mathematical induction, prove that $\sum_{r=1}^{n}\left(2^{r-1}-1\right)=2^{n}-n-1$ for all $n \in Z^{+}$

02. Sketch the graph of $y=|x+2|$ and $y=4-2|x-1|$ in the same diagram. **Hence or otherwise**, find all real values of x satisfying the inequality $|x+2|+2|x-1| \geq 4$.

Deduce, the range of real values of x , which satisfies the inequality $|x|+2|x-3| \geq 4$.

03. On an Argand diagram, sketch the loci of the points representing complex numbers w and z such that $|w-1-2i|=1$ and $\arg(z-1)=\frac{3}{4}\pi$. Find the least value of $|w-z|$ for the points on these loci.

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04 . Find the coefficient of x^7 in the expansion of $(1-x^4)(1+x)^9$.

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05. Show that $\lim_{x \to 0} \frac{1-\cos x\sqrt{\cos 2x}}{x^2} = \frac{3}{2}$.

06. Find the volume generated by rotating the region enclosed by the curves $y = \sqrt{x}$, $y = 6 - x$ and the x - axis, about the x - axis.

07. The parametric equations of a curve are given by $x = t^3 - 8t$, $y = t^2$, where t is a parameter. Given that the point A has parameter $t = -1$. Find the equation of the tangent drawn to the curve at the point A. The tangent also intersects the curve at the point B. Find the coordinate of the point B.

08. Triangle ABC has vertices $(0,0)$, $(11,60)$ and $(91,0)$. If the line $y = kx$ divides the triangle in to two triangles of equal area, then find the value of k.

09. Find the equation of the in-circle of the triangle formed by the coordinate axes and the line $4x+3y=6$.

10. Solve the equation $(1-\tan\theta)(1+\tan\theta)\sec^2\theta+2^{\tan^2\theta}=0$

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Provincial Department of Education - NWP

තෙවන වාර පරීක්ෂණය - 13 ශ්‍රේණිය - 2024
Third Tem Test - Grade 13 - 2024

Combined Mathematics - I

Time: 03 Hours

Part B

❖ ***Answer only five questions.***

11. *a.* The roots of the quadratic equation $x^2 - x + p = 0$ are α, β and those of $x^2 - 4x + q = 0$ are γ, δ.

If $\alpha, \beta, \gamma, \delta$ are four consecutive terms of a geometric progression whose common ratio is r, then show that $r = \pm 2$.

Find all possible values of p and q.

If $\alpha < \beta$ and $\gamma < \delta$, when p and q takes above possible values, find the quadratic equation whose roots are $\alpha\gamma$ and $\beta\delta$.

b. Let $9x^2 + 6x + 1 = 4kx; \quad k \in R$. Find the set of values of k such that the roots of this equation are real and positive.

c. A polynomial $f(x)$ is defined in terms of a constant k by $f(x) = x^3 + kx^2 - x + 12$.

When $f(x)$ is divided by $(x-1)$, the remainder is r and when $f(x)$ is divided by $(x-4)$, the remainder is $8r$.

i. Find the values of k and r.

ii. Show that $(x+4)$ is a factor of $f(x)$.

iii. Show also that $f(x) = 0$ has only one real root.

12. *a.* Find the number of permutations that can be made from the letters of the word **PUTHUKKUDIYIRUPPPU** taking **four** letters at a time.

b. Write down u_r the r^{th} term, of the series $\frac{8}{2\cdot 5}\left(\frac{1}{2}\right) + \frac{11}{5\cdot 8}\left(\frac{1}{2}\right)^2 + \frac{14}{8\cdot 11}\left(\frac{1}{2}\right)^3 + \ldots\ldots\ldots\ldots$

Find the values of the constants A and B such that $\frac{3r+5}{(3r-1)(3r+2)} = \frac{A}{(3r-1)} - \frac{B}{(3r+2)}$ for $r \in Z^+$.

Hence or Otherwise, find a function $f(r)$ such that $Ur = f(r) - f(r-1)$.

Hence, evaluate $\sum_{r=1}^{n} u_r$.

Is the infinite series $\sum_{r=1}^{\infty} U_r$ convergent? Justify your answer.

Further show that $\frac{2}{5} \leq \sum_{r=1}^{n} u_r \leq \frac{1}{2}$

13. *a.* If $A = \begin{pmatrix} 2 & a \\ 3 & b \end{pmatrix}$, $B = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $C = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$, find the values of a and b such that $AB = C$.

For above values of a and b, show that $A^2 = 2A - 3I$.

The inverse of A is denoted by A^{-1} and I is the 2×2 identity matrix. Show further that

i. $A^3 = A - 6I$ ii. $A^{-1} = \frac{1}{3}(2I - A)$

Hence, Find A^{-1}

b. The complex numbers $-1+3i$ and $2-i$ are denoted by u and v respectively. In an Argand diagram with origin O, the points A, B and C represent the numbers u, v and $v+u$ respectively.

Sketch this diagram and state the geometrical relationship between OB and AC .

Prove that angle $A\hat{O}B = \frac{3}{4}\pi$.

c. The complex number w is defined by $w = \frac{22+4i}{(2-i)^2}$. show that $w = 2+4i$.

It is given that a is a real number such $\frac{1}{4}\pi \leq \arg(w+a) \leq \frac{3}{4}\pi$. Find the set of possible values of a .

The complex conjugate of w is denoted by $\overline{w}$. The complex numbers w and $\overline{w}$ are represented in Argand diagram by the points P and Q respectively. Find the equation of circle passing through P, Q and the origin in the form $|z-b| = k$.

14. *a.* Let $f(x) = \frac{2x-3}{(x-1)^2}$ for $x \neq 1$.

Show that $f'(x)$, the derivative of $f(x)$, is given by $f'(x) = \frac{-2(x-2)}{(x-1)^3}$ for $x \neq 1$.

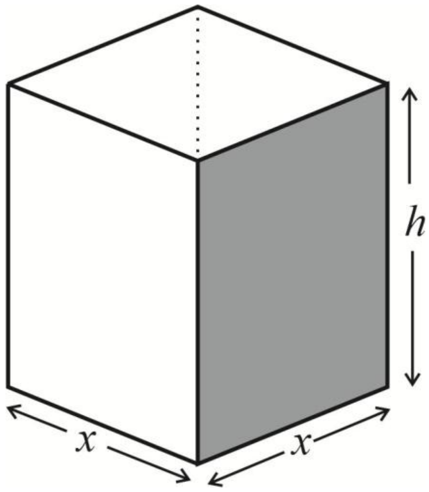
Hence find the interval on which $f(x)$ is increasing and the intervals on which $f(x)$ is decreasing. Also find the coordinates of the turning point of $f(x)$.

It is given that $f''(x)=\dfrac{2(2x-5)}{(x-1)^4}$ for $x\neq 1$. Find the coordinates of the inflection point.

Sketch the graph of for $y=f(x)$ indicating the asymptotes, the turning points, y - intercept and the point of inflection.

b. The figure shows the design of a large water tank in the shape of a cuboid with a square base and no top.

The square base is of length x metres and height h metres. It is given that the volume of the tank is $500\, m^3$.



i. Show that the surface area of the tank, $A\, m^2$, is given by $A=x^2+\dfrac{2000}{x}$

ii. Find the value of x such that the surface area is minimum and find the minimum value of A.

15. *a.* Express $\dfrac{1}{(x+1)^2(x+2)}$ in partial fractions and **Hence**, Find $\displaystyle\int\frac{1}{(x+1)^2(x+2)}dx$

Using a suitable substitution or Otherwise, Find $\displaystyle\int\frac{\cos x}{(1+\sin x)^2(2+\sin x)}dx$.

b. By applying integration by parts, Show that $\displaystyle\int_0^{\pi} x\cos\left(\frac{1}{4}x\right)dx=2\sqrt{2}(\pi+4)-16$.

c. Show that $\displaystyle\int_0^a f(x)\,dx=\int_0^a f(a-x)\,dx$.

If $I=\displaystyle\int_0^{\frac{\pi}{4}}\ln|1+\tan\theta|\,dx$, using the above result, show that $\displaystyle\int_0^{\frac{\pi}{4}}\ln|1+\tan\theta|\,dx=\frac{\pi}{8}\ln 2$.

d. Find A and B such that $2\sin x+3\cos x\equiv A(3\sin x+4\cos x)+B(3\cos x-4\sin x)$

Hence, find $\displaystyle\int\frac{2\sin x+3\cos x}{3\sin x+4\cos x}dx$.

16. The equations of two circles are given as $x^2+y^2+2g_1x+2f_1y+c_1=0$ and $x^2+y^2+2g_2x+2f_2y+c_2=0$. If these two circles intersect each other orthogonally, show that $2g_1g_2+2f_1f_2=c_1+c_2$.

Let $S_1 \equiv x^2+y^2-4x-2y+4=0$ and $S_2 \equiv x^2+y^2-12x-8y+36=0$.

Show that above two circles touching each other.

If that point is M and find its coordinates of M.

Find the equation of the common tangent passing through M.

Also find the equations of other two common tangents of the circles.

If the touching points of these common tangents and S_2 are A and B, and the touching points of these common tangents and S_1 are C and D find the coordinates of A, B, C and D.

Obtain the coordinates of the intersection point N of the line passing through the two centres of the two circles and the x - axis.

Find the equation of the circle whose centre is N and passing through A or B.

Show that the above obtained circle and S_2 intersect orthogonally.

17. *a*. Starting from the formulae for $\sin(A+B)$ and $\cos(A+B)$, prove that

$$\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$$

Deduce that $\tan\left(\theta+\frac{\pi}{3}\right)=\frac{\sqrt{3}+\tan\theta}{1-\sqrt{3}\tan\theta}$. **Hence or otherwise**, solve the equation $\sqrt{3}+\tan\theta=\left(1-\sqrt{3}\tan\theta\right)\tan(\pi-\theta)$ in the interval $0\le\theta\le\pi$.

b. State and prove the ***Cosine rule*** for a triangle ABC, in the usual notation.

Let ABC be a triangle. In the usual notation, it is given that D is the midpoint of the side BC. If AD is perpendicular to AC, then show that

$$\cos A\cos C=\frac{2\left(c^2-a^2\right)}{3ca}.$$

c. Show that, $\sin^{-1}x-\cos^{-1}x=\sin^{-1}(3x-2)$ for $\frac{1}{3}\le x\le 1$.

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Third Term Test - Grade 13 - 2024

විභාග අංකය: ....

Combined Maths - II

කාලය පැය 03 යි
අමතර කියවීම් කාලය - මිනිත්තු 10 යි

උපදෙස්

- මෙම ප්‍රශ්න පත්‍රය කොටස් දෙකකින් සමන්විත වේ.
  A කොටස (ප්‍රශ්න 1-10) දක්වා B කොටස (ප්‍රශ්න 11-17)
- A කොටස
  සියලුම ප්‍රශ්නවලට පිළිතුරු සපයන්න. එක් එක් ප්‍රශ්නය සඳහා ඔබේ පිළිතුරු සපයා ඇති ඉඩෙහි ලියන්න.
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- B කොටස
  ප්‍රශ්න පහකට පමණක් පිළිතුරු සපයන්න.
- නියමිත කාලය අවසන් වූ පසු A කොටස B කොටසට උඩින් සිටින පරිදි කොටස් දෙක අමුණා විභාග ශාලාධිපතිට භාර දෙන්න.
- ප්‍රශ්න පත්‍රයෙහි B කොටස පමණක් විභාග ශාලාවෙන් පිටතට ගෙනයාමට ඔබට අවසර ඇත.

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B	එකතුව	
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පත්‍රය I	
පත්‍රය II	
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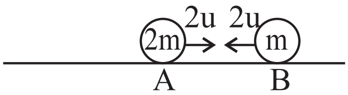
	2	
අධීක්ෂණය		

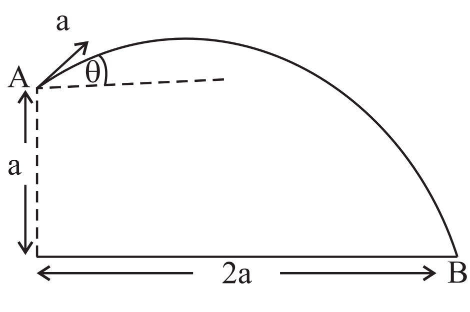
PART - A

(01) A particle A of mass **2m** and a particle B of mass **m** moving on a smooth horizontal table along the same straight line towards each other with speeds 2u collide direcly. The coefficient of restitution between A and B is **e**.

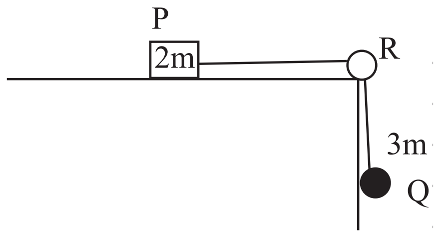


Find the velocities of A and B after the collision. If A comes to rest after the collision show that $\mathbf{e} = \frac{1}{2}$.

(02) A particle is projected from a point A, at a vertical distance **a** above a horizontal ground with initial velocity **u**, at an angle $\theta = \tan^{-1}\left(\frac{3}{2}\right)$ to the horizontal. The particle strikes the ground at a horizontal distance **2a** from A. (see the figure) show that $\mathbf{u} = \sqrt{\frac{13ga}{8}}$

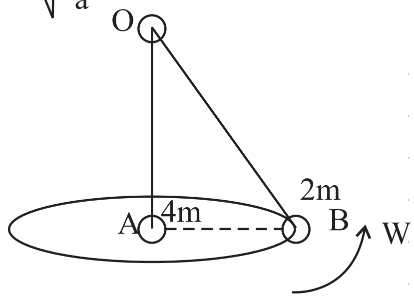


(03) Two particles P and Q of masses **2m** and **3m** respectively, are attached to the ends of a light inextensible string. The particle P is held at rest on a rough horizontal table with the string passing over a, small smooth pulley fixed at the edge of the table. The particle Q hangs vertically below the pulley. The system is released from rest with the string taut. If the coefficient of friction between the particle P and the table is $\frac{1}{3}$, write down the equations sufficient to determine the tension of the sring.



(04) A van of mass 2000 kg moves upwards with a constant acceleration along a straight road of inclination $30^{0}C$ to the horizontal. There is a constant resistance of 400 N to its motion. Find the acceleration of the van when it moves with speed $20ms^{-1}$ and when the engine of it is working with a power of 60 kW.

(05) Two particles A and B masses **4m** and **2m** are attached to two ends of string of length ***l***, passing through a fixed smooth ring O. Particle A is at rest vertically below O. When the particle A is at rest, the particle B moves in a horizontal circle with A as the centre. (See the figure). Show that $OA = \frac{l}{3}$ and if the angular velocity of B is ω, also show that $\omega = 2\sqrt{\frac{g}{a}}$.

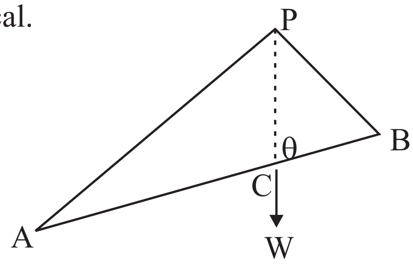


(06) Let $\underline{a} = \underline{i} + 3\underline{j}$ and $\underline{b} = \lambda\underline{i} + \mu\underline{j}$. Here λ and μ are real constant and $\mu>0$. Also $|\underline{b}| = 2$ and the two vectors $\underline{a}$ and $\underline{b}$ are perpendicular to each other. If $\underline{i}$ and $\underline{j}$ are unit vectors as usual, find the values of λ and μ .

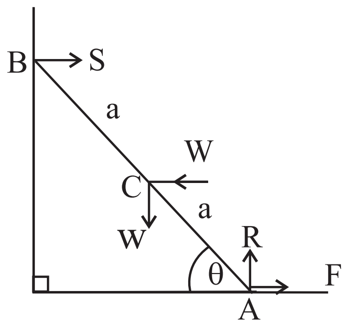
(07) The centre of gravity of a non uniform rod AB of weight W is in the ratio 3 : 2. It is hung in a vertical plane by a string attached to the two ends A and B, which passes above a small smooth peg P. Then the rod makes an angle $\theta = \tan^{-1}\sqrt{2}$ with the upward vertical.

(i) Show that, $A\hat{P}C = B\hat{P}C$.

(ii) Also show that, $A\hat{P}C = \tan^{-1}\dfrac{\sqrt{2}}{5}$



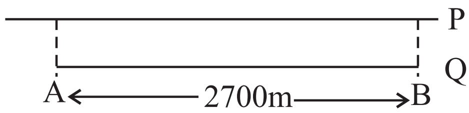
(08) An uniform rod AB of weight **W** and length **2a** is kept in a vertical plane perpendicular to the wall with its lower end A is on a rough horizontal ground with coefficient of friction $\frac{1}{3}$, and end B against a smooth vertical wall, by a horizontal force of magnitude **W** applied at the mid point of the rod towards the wall as shown in the figure. The frictional force and the normal reaction at A are **F** and **R** respectively and the normal reaction on the rod by the wall at B is **S**. If the rod is in limiting equilibrium, show that the angle θ which the rod makes with the horizontal is $\theta = \tan^{-1} 3$.



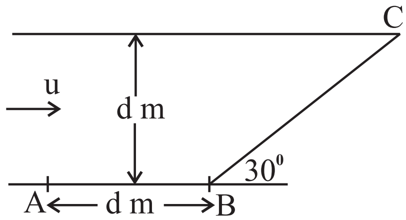
(09) Let A and B be two events of a sample space Ω. In the usual notation, it is given that $P(B) = \frac{1}{2}$, $P(A|B) = \frac{2}{5}$ and $P(A \cap B') = \frac{1}{10}$ Find $P(A \cap B)$, $P(A)$ and $P(A \cup B)$.

(10) In the set of data $\{x_1, x_2, x_3, ... x_{10}\}$, Let $\sum_{i=1}^{10} x_i = 200$ and $\sum_{i=1}^{10} (x_i - 10)^2 = 1050$. Find the mean and the variance of the above data set.

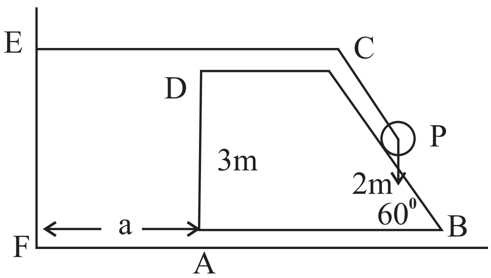
(11) (a) P and Q are two straight parallel train tracks between two railway stations A and B that are 2700 m apart. A train X that has a velocity V ms^{-1} at t = 0 at station A travels along the track P with an uniform retardation $5ms^{-2}$ for T seconds and obtains a velocity 10 ms^{-1} at t = T s and then maintains that speed for 30 seconds. Then it moves with uniform acceleration of $5ms^{-2}$ for T seconds and obtains a velocity V ms^{-1} at the station B. Sketch the velocity - time graph for the motion of the train X from A to B and show that the total time taken for the journey is 70 S. Another train Y travelling along the track Q in the direction AB at a constant speed of $50ms^{-1}$ passed the station A at t = 0. Sketch the velocity - time graph for the motion of the train X relation to the train Y from t = 0 to t = 70 s.



(b) A river **d** m wide, flows at a uniform speed **u** ms^{-1}, in between two straight parallel banks. A swimmer P whose speed relative to water is $\sqrt{3}$ **u** ms^{-1} , starts from a point A on one bank and swims a distance **d** m down stream and then swim to a point C in the opposite bank which makes an angle 30^0 with the river bank. (see the figure). Show that the total time taken for the swimmer to swim from A to the point C through B is $\left(\frac{1}{\sqrt{3}+1} + \frac{2}{\sqrt{7}}\right)\frac{d}{u}$.



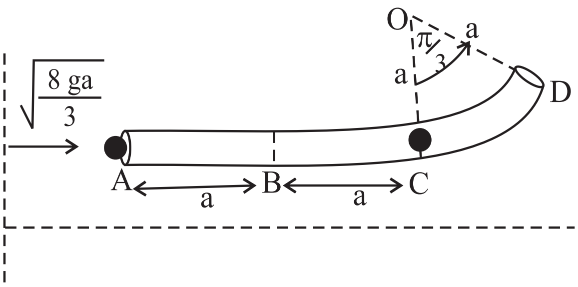
(12) (a) The trapezium ABCD in the figure is the vertical cross section through the centre of gravity of a smooth uniform block of mass 3m with $A\hat{B}C = \frac{\pi}{3}$ Such that the face containing AB is placed on a smooth horizontal floor. The lines AB and DC are parallel and the line BC is a line of greatest slope of the face containing it. One end of a light inextensible string passing over a small smooth pulley fixed at C is attached to a particle P of mass 2 m and the other end is attached to the fixed point E on the wall. Then the system is released from rest such that the string is taut with the particle P held on BC and the distance to the end A of the block from the wall is **a** initially as shown in the figure.



Let the acceleration of the block relation to the floor towards the wall is F and the acceleration of the particle P relative to the block towards CB is F and show that f = F.

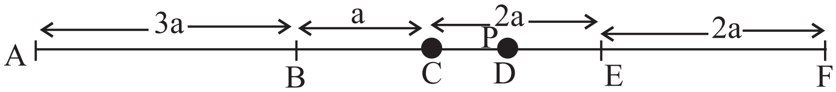
Also show that, $F = \frac{\sqrt{3}\ g}{5}$ and the time taken for the block to reach the wall is $\sqrt{\frac{10\ a}{\sqrt{3}\ g}}$

(b) A thin tube ABCD is fixed in a vertical plane with ABC horizontal, as shown in the figure. Each of the sections AB and BC is of length **a** and the section CD is a part of a circle of radius **a** and centre O, with OC vertical such that $C\hat{O}D = \frac{\pi}{3}$. A particle Q of mass **m** is placed inside the tube at the point C. Another particle P of mass **m** is placed inside the tube at the point A, and it is given a velocity of magnitude



$\sqrt{\frac{8\,ga}{3}}$ in the direction of $\overrightarrow{AB}$. The coefficient of friction between the particle P and the section AB is $\frac{1}{3}$, and the section BCD is smooth. The particle P moves through the tube, and collides and coalesces with the particle Q. Show that the velocity with which this combined particle R starts to move is $\sqrt{\frac{ga}{2}}$. Show that the speed V of the particle R, when $\overrightarrow{OR}$ turns through an angle θ $(O < \theta < \frac{\pi}{3})$ with the downward vertical is given by, $V^2 = \frac{ga}{2}(4\cos\theta - 3)$ and also show that the reaction S on the particle R from the tube at that instant is given by $S = 3\,mg\,(2\cos\theta - 1)$. Also show that the particle R will not reach the point D.

(13)



The points A, B, C, D, E and F lie on a stralght line in that order, on a smooth horizontal table such that AB = 3a, BC = a, CE = 2a and EF = 2a as shown in the figure. One end of a light elastic string of natural length 3a and modulus of elasticity 2mg is attached to the point A and the other end to a particle P of mass m. One end of another light elastic string of natural length 2a and modulus of elasticity mg is attached to the point F and the other end to the particle P. If the particle P stays in equilibrium at D, show that $AD = \frac{30\,a}{7}$.
Now the string AP is pulled until the particle P reaches the point E and released from rest. Show that the equation of motion of P from E to B is given by $\ddot{X} + \frac{7g}{6a}\left(x - \frac{30a}{7}\right) = 0$. Where $AP = x$. Considering $X = x - \frac{30a}{7}$, show that $\ddot{X} + \frac{7g}{6a}x = 0$.
Find the centre of the motion, Using the formula $\dot{X}^2 = \frac{7g}{6a}(C^2 - X^2)$, where C is the amplitude, find the value of C and show that the Velocity of the particle P when it reaches B is . $\sqrt{\frac{3ga}{2}}$

Show that the time taken by the particle P started at E, to reach B is $\sqrt{\frac{6a}{7g}}\left[\pi - \cos^{-1}\left(\frac{3}{4}\right)\right]$.

(14) (a) In the usual notation, Lets $\underline{a} = 2\underline{i} + 3\underline{j}$, $\underline{b} = -\underline{i} - 2\underline{j}$, $\underline{c} = 5\underline{i} - \underline{j}$ and $\underline{d} = 5\underline{i} + \underline{k}\underline{j}$ where $K \in \mathbb{R}$,

be the position vectors of four points A, B, C and D respectively, with respect to a fixed origin O. The lines AD and BC are parallel. Show that $K = \frac{7}{2}$.

The lines AC and BD intersect at the point P where the position vector is $\underline{P}$.

Considering $\overrightarrow{AP}$ and $\overrightarrow{BP}$ shown that $\underline{P} = \underline{a} + (\underline{c} - \underline{a})$ for $\lambda \in \mathbb{R}$ and $\underline{P} = (1 - \mu)\,\underline{b} + \mu\underline{d}$ for $\mu \in \mathbb{R}$.

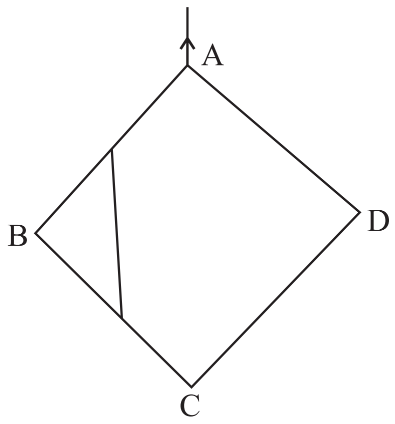
Hence show that $\lambda = \frac{1}{3}$ and $\mu = \frac{2}{3}$ and find $\underline{P}$ in terms of $\underline{i}$ and $\underline{j}$.

(b) A system of forces in the Oxy plane consists of three forces $-3P\underline{i} + 4P\underline{j}$, $5P\underline{i} - 2P\underline{j}$ and $6P\underline{i} + 4P\underline{j}$ acting at the points $A \equiv (-2a, 0)$, $B \equiv (a, a)$ and $C \equiv (a, 2a)$ respectively, where P and a are positive quantities measured in newtons and metres, respectively. Show that the clockwise moment of the system about the origin O is 23 Pa Nm.

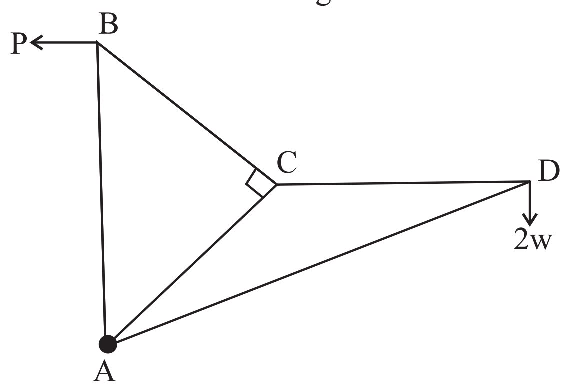
Also, show that the system is equivalent to a single resultant force of magnitude 10p N and find its direction and show that the equation of the line of action is $8y - 6x - 23a = 0$.

Now, an extra force is introduced to the system so that the new system is equivalent to a couple with clockwise moment 30Pa Nm. Find the magnitude, direction and the line of action of the extra force.

(15) (a) Two uniform rods AB and AD each of length **2a** and weight **2W** and another two uniform rods BC and CD each of length **2a** and weight **W** are smoothly jointed at their ends A, B, C and D to form a square ABCD. The mid points of AB and BC are joined by a light inextensible string. The system is suspended in a vertical plane from the point A and stays in equilibrium as show in the figure. Show that the magnitude of the reaction exerted on BC by CD at the joint C is $\frac{\sqrt{34}}{4}w$ and find its direction. Also show that the tension in the string is 5W.



(b) The framework shown in the figure consists of five light rods AB, BC, AC, CD and AD smoothly jointed at their ends. It is given that $AC = CD = 2a$, $A\hat{C}B = 90^0$, $C\hat{A}D = B\hat{A}C = 30^0$. At the joint D, a load 2W is suspended and the framework is smoothly hinged at A to a fixed point and kept in equilibrium in a vertical plane with AB vertical and CD horizontal, by a horizontal force P applied to it at the joint B.



(i) Show that $P = \frac{3\sqrt{3}w}{2}$

(ii) Draw the stress diagram using Bow's notation for the joints B, C and D. Hence, find the stresses in the rods, stating whether they are tensions or thrusts.

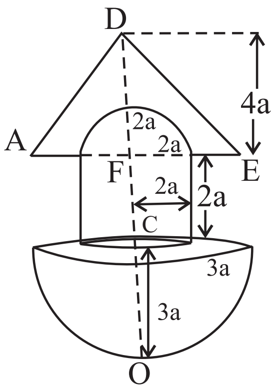
(16) Show that the centre of mass of

(i) a uniform solid right circular cone of base radius ***r*** and height ***h*** is at a distance $\frac{h}{4}$ from the centre of the base.

(ii) a uniform solid hemisphere of radius ***r*** is at a distance $\frac{3r}{8}$ from its centre.

A uniform solid hemisphere of radius **2a** and centre F is carved out from a solid right circular cone of radius **3a**, height **4a** and centre F, and the composite body R is made by rigidly fixing this and a solid cylinder of radius of the base **2a** and height **2a** to a uniform solid hemisphere of radius **3a** and centre C as shown in the figure.

Show that the centre of mass of the composite body R lies at a distance Ka from O, Here $K = \frac{1285}{392}$. The composite body R is Freely suspended by a vertical string attached to the point A of the cone, and hangs in equilibrium such that OF making an angle θ with the downward vertical. Show that $\tan\theta = \frac{3}{5 - k}$



(17) (a) A and B are two identical boxes. Box A contains 3 red balls and 7 blue balls while box B contains 4 red balls and 6 blue balls. The balls are identical in all aspects, except for their colour. Now, two unbiased regular tetrahedron with faces numbered 1, 2, 3 and 4 are rolled together. If the sum of the numbers then obtained is a multiple of 4, box A is chosen; if not box B is chosen. From the bag chosen two balls are drawn one after the other at random without replacement.

(i) Find the probability that both balls drawn are red.

(ii) Given that both balls drown are red, find the probability that these balls were drawn from box A.

(b) The marks of 100 students for a certain test is given in the following table.

Marks	Number of students
0 - 20	10
20 - 40	20
40 - 60	30
60 - 80	30
80 - 100	10

Estimate the median, mean and the Variance of the frequency distribution given above.

Later marks of another 50 students fell in to each time interval in the above table, 10 students each.

Estimate the mean of the new distribution.