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மேல் மாகாணக் கல்வித் திணைக்களம்
Department of Education - Western Province

සංයුක්ත ගණිතය I
இணைந்த கணிதம் I
Combined Mathematics I

10 S I

පැය තුනයි

மூன்று மணித்தியாலம்
Three hours

අමතර කියවීමේ කාලය

மேலதிக வாசிப்பு நேரம்
Additional Reading Time

- මිනිත්තු 10 යි

- 10 நிமிடங்கள்
- 10 minutes

Use additional reading time to go through the question paper, select the questions you will answer and decide which of them you will prioritise.

Index number :

Instructions :

- * This paper consists two parts: Part A (questions 1-10) and Part B (questions 11-17)
- * **Part A :**
Answer all questions. Write your answers to each questions in the space provided. You may use additional sheets if more space is needed.
- * **Part B :**
Answer **five** questions only. Write your answers on the sheets provided.
- * At the end of the time allocated, tie the answer script of the two parts together so that **part A** is on the top of **part B** and hand them over to the supervisor.
- * You are permitted to remove **only part B** of the question paper from the examination hall.

Examiner's use only

(10) Combined Mathematics I		
Part	Question No	Marks
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	
Total		

Total

In Number	
In words	

Code numbers

Marking examiner	
Checked by	1
	2
Supervised by	

Part A

- 1) Using the **principle of mathematical induction** prove that, $(2x + 1)^n - (x - 3)^n$ is divisible by $(x + 4)$ for $n \in \mathbb{Z}^+, x \in \mathbb{R}$.

[illegible]

- 2) Sketch the graph of $y = 3 - 2|x - 1|$, and $y = |2x + 1|$ in the same diagram. **Hence or otherwise** for all real values of x satisfying the inequality $|4x + 1| \leq 3 - 2|2x - 1|$.

[illegible]

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- This image shows a full page of primary-ruled paper. It features multiple horizontal rows of small black dots, evenly spaced across the entire page. These rows are designed to guide young learners as they practice writing letters and words. The background is white, and there are no margins or other markings present.

- [illegible]

[illegible][illegible]

[illegible]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting or typing. There are no margins, text, or other markings on the page.

- [illegible]

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Part B

* Answer only 5 questions.

- 11) (a) Show that the quadratic equation $x^2 + px + p = 0$ has real roots, when $p \geq 4$ only where $p \in \mathbb{R}$. If the roots of the above equation are α, β write the values of $(\alpha + \beta), \alpha\beta$ and show that $\left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right) = \frac{(p+1)^2}{p}$. Show that the quadratic equation whose roots $\left(\alpha + \frac{1}{\beta}\right), \left(\beta + \frac{1}{\alpha}\right)$ is given by $px^2 + p(p+1)x + (p+1)^2 = 0$ and deduce the quadratic equation whose roots are $\left(\alpha + \frac{1}{\beta}\right)^2, \left(\beta + \frac{1}{\alpha}\right)^2$.
- (b) Let $f(x) = x^5 + 80x^2 + 240x + 192$ Using remainder theorem show that $(x+2)^3$ is a factor of $f(x)$. Show also that $f(x) = 0$ has only one real root.
- 12) (a) Given that $(bx + 2a)^8 = C_0 + C_1x + C_2x^2 + \dots + C_i x^i + \dots + C_8x^8$ where a and b are constants. The coefficient of x_i is C_i where $i = 0, 1, 2, \dots, 8$. If $7aC_1 = 2C_2$ show that $b = 2a^2$. Hence or otherwise show that $\sum_{i=1}^8 C_i = (2a)^8[(a+1)^8 - 1]$.
- (b) Write r^{th} term T_r of the sequence $\frac{8}{2.5} \cdot \left(\frac{1}{2}\right) + \frac{11}{5.8} \cdot \left(\frac{1}{2}\right)^2 + \frac{14}{8.11} \cdot \left(\frac{1}{2}\right)^3 + \dots$ and show that $T_r = f(r) - f(r+1)$ where $f(r) = \frac{1}{(3r-1)} \left(\frac{1}{2}\right)^{r-1}$
- Hence show that $\sum_{r=1}^n T_r = \frac{1}{2} - \frac{1}{(3n+1)} \left(\frac{1}{2}\right)^n$ and the sequence is convergent. Further show that $\frac{2}{5} \leq \sum_{r=1}^{\infty} T_r \leq \frac{1}{2}$.
- 13) (a) Given that $A = \begin{pmatrix} 2 & 1 \\ 0 & -1 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 0 & -2 \end{pmatrix}$ Find the values of the constants λ, μ such that $A^2 + \lambda A + \mu I = 0$ I is a 2×2 unit matrix. Show that $B^{-1} = \frac{1}{2}A$ and find A^{-1} .
- (b) Let $Z = x + iy$ where $x, y \in \mathbb{R}$. Write $\bar{Z}$ the conjugate of Z and $|Z|$. Show that
- $Z \cdot \bar{Z} = |Z|^2$
 - $Z - \bar{Z} = 2i \operatorname{Im}(Z)$
- Also show that, when $|Z| = 1$ $\frac{Z-1}{Z+1}$ is a pure imaginary number.

- (c) Find the square roots P and Q of the complex number $12 + 5i$ in the form $a + ib$, ($a, b \in \mathbb{R}$) and represent P and Q on an Argand diagram. If Q' is the conjugate complex of Q, find the complex number R such that OPRQ' is a rhombus.

14) (a) Let $f(x) = \frac{x^2+4}{(x-2)^2}$, for $x \neq 2$. show that $f'(x)$ the derivative of $f(x)$ is given by $f'(x) = \frac{-4(x+2)}{(x-2)^3}$

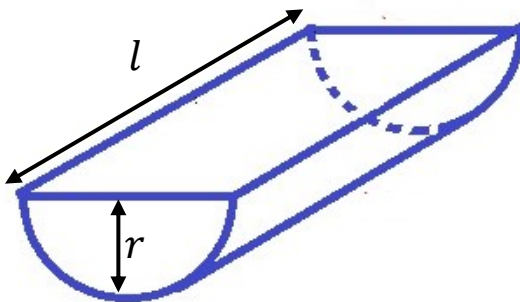
for $x \neq 2$. Find the coordinates of the turning point of $f(x)$.

Hence find the interval of which $y = f(x)$ is increasing and the interval of which $y = f(x)$ is decreasing.

Given that $f''(x) = \frac{8(x+4)}{(x-2)^4}$, $x \neq 2$ find the coordinates of the point of inflection of the graph $y = f(x)$

Sketch the graph of $y = f(x)$ indicating the asymptotes, the turning point and the point of inflection.

- (b) The diagram shown below is a semi cylindrical container of radius r cm and length l cm. If the volume of the container is $512\pi \text{ cm}^3$ and the surface area is $S \text{ cm}^2$ show that S is given by $S = \pi r \left(r + \frac{1024}{r^2} \right)$ Also find the value of r when S is minimum and find the minimum surface area.



15) (a) Find $\int \frac{3}{(x-1)(x^2+x-2)} dx$ using partial fractions.

If $I = \int_0^{\frac{\pi}{6}} \frac{\cos x}{(\sin x - 1)(\sin x + 2)} dx$ show that, $I = \frac{1}{3} \ln \left(\frac{5}{2} \right)$

- (b) Integration by parts and suitable substitution, show that,

$$\int_0^{\frac{\pi}{3}} \tan^{-1}(\sin x) \cdot \cos x dx = \frac{2\pi-3}{2\sqrt{3}}$$

- (c) Show that $\int_0^a f(x) dx = \int_0^a f(a-x) dx$ where a is a constant

Let $I = \int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{\sin x + \cos x} dx$. Using the above results show that,

$$I = \frac{1}{2} \int_0^{\frac{\pi}{2}} \frac{1}{\sin x + \cos x} dx. \text{ Also show that } I = \frac{1}{\sqrt{2}} \ln(\sqrt{2} + 1).$$

16) In a parallelogram $ABCD$, $AD \equiv x + y = 12$, $AB \equiv y = 2x$, $B \equiv (0, \lambda)$ and $C \equiv (-2\sqrt{10}, \mu)$. Find λ , and μ . Show that any point on AC can be written in the form $(4 + t, 8 + (\sqrt{10} - 3)t)$ where t is a parameter. Show that the circle which touches BC and the center lies on AC is given by $x^2 + y^2 - 2(t + 4)x - 2(8 + (\sqrt{10} - 3)t)y + c = 0$ Where $c \in \mathbb{R}$. When $c = t^2(12 - 4\sqrt{10}) + t(4\sqrt{10} - 16) + 8$ if the circle touches AB show that $t = -(\sqrt{10} + 2)$. Find the coordinate of the center of this circle and show that the center is the point of intersection of the diagonals of the parallelogram. Show also that this circle touches CD and find the equations of the circles touch internally and externally at B .

17) (a) Express $\sin x + \sqrt{3} \cos x$ in the form $R \sin(x + \alpha)$ where $R > 0$, $0 < \alpha < \frac{\pi}{2}$.

If $\sec x + \sqrt{3} \operatorname{cosec} x = 4$, for $0 \leq x \leq \pi$, show that

$\sin 2x - \sin\left(x + \frac{\pi}{3}\right) = 0$. Hence or otherwise find the general solution of

$$\sec x + \sqrt{3} \operatorname{cosec} x = 4$$

(b) In a triangle ABC angle $B = \frac{\pi}{2}$. O is a point inside the triangle which makes an angle $\frac{2\pi}{3}$ with one side of the triangle. If $\angle CBO = \theta$ show that, $\tan \theta = \frac{c + a\sqrt{3}}{a + c\sqrt{3}}$.

(c) Solve the equation $\tan^{-1}\left(\frac{x+1}{x+2}\right) + \tan^{-1}\left(\frac{x-1}{x-2}\right) = \frac{\pi}{4}$.



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